Parallel Multilevel Iterative Linear Solvers with Unstructured Adaptive Grids for Simulations in Earth Science

Kengo Nakajima

Department of Computational Earth Sciences, Research Organization for Information Science and Technology (RIST), Tokyo, Japan (e-mail: nakajima@tokyo.rist.or.jp, phone: +81-3-3712-5321, fax: +81-3-3712-5552).

Abstract

In many large-scale scientific simulation codes, the majority of computation is devoted to linear solvers. Preconditioned Krylov iterative solver such as conjugate gradient method with incomplete Cholesky factorization preconditioning (ICCG) provides robust convergence for a wide range of scientific applications. Incomplete Cholesky (IC) and incomplete LU (ILU) factorizations involve globally dependent operations, yet can be localized for parallel computation and provide smooth convergence. However, ICCG-based solvers are not scalable because the number of iterations required for convergence increases with the size of the problem. This becomes critical when solving problems with $>10^9$ degrees of freedom (DOF) on $>10^4$ processors.

Multigrid is a well-known scalable method for which the rate of convergence is independent of problem size, and the number of iterations remains fairly constant. Multigrid is also a good preconditioning algorithm for Krylov iterative solvers. Multigrid solvers and preconditioners have been widely used in finite-difference methods with structured grids since the mid 1980s, however multigrid is not popular for finite-element methods involving unstructured grids. Recently, various multigrid methods for unstructured grids have been developed, for both parallel and serial computers.

In this study, a multigrid-preconditioned conjugate gradient iterative method (MGCG) on parallel computers was developed and applied to Poisson equations in the region between 2 spherical surfaces on adaptively generated semi-unstructured prismatic grids based on the method in [1]. This procedure has also been applied to grids with local refinement. Computations using this method on a Hitachi SR2201 with up to 128 processors demonstrated good scalability.

![Fig. 1] Surface triangle meshes generated from icosahedron
4 children generated from 1 parent triangle
Fig. 2  Multilevel communication table

(a) Uniform boundary condition

(b) Single-patch boundary condition

Fig. 3  Computation time (including communication for parallel computing) for fixed problem size on each processor (320×900=288,000 cells/PE) for 2 to 32 PEs (up to 9,216,000 cells). Black squares: ICCG, black circles: MGCG/GS, white Circles: MGCG/ILU(0).

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References