Damage Localization, Sensitivity of Energy Release and Catastrophe Transition

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Abstract

Large earthquakes can be viewed as catastrophic ruptures in the earth's crust. There are two common features prior to catastrophe transitions in heterogeneous media. One is damage localization and the other is critical sensitivity, both of them are related to cascade of damage coalescence. These may be two cross checking precursors of large earthquakes.

Introduction

Generally speaking, the main rupture in earthquakes result from a large-size unstable cascade of damage coalescence beyond a critical transition in a certain region. Large earthquakes are preceded by an increase of intermediate-sized events over a large area [1-4]. From the viewpoint of rock mechanics, there are two outstanding features in rupture of heterogeneous media, like earthquakes in crust, [5]. These are fracture surface formation and rapid release of stored elastic energy. On one hand, fracture surface formation is preceded by a process of damage localization in heterogeneous media. On the other hand, the release of stored energy corresponding to cracking may imply some omen of main rupture. So, predictions should concern this sort of precursors ahead of the catastrophe transition. Damage localization and sensitivity related to energy release may be two cross checking precursors of the catastrophe transition, according to the above-mentioned essence of earthquakes.

Earthquakes usually occur in a seismic area loaded by surrounding crust. For a body loaded from its surrounding, its catastrophe transition depends on its increment of dissipation energy

$$\Delta W = F \cdot \Delta u = F \cdot (\Delta u_b + \Delta u_s) = F \cdot \Delta F \cdot (1/K_b + 1/K_s) < 0, \qquad (1)$$

where F is force and u is displacement, u_b , K_b and u_s and K_s are the displacements and the current stiffness of the body and its surrounding respectively. If the force F and the surrounding's stiffness K_s are always positive, the body would become unstable, when

$$K_b < -K_s \qquad \text{provided } \Delta F < 0 \text{ and } K_b < 0. \qquad (2)$$

So, the catastrophe transition of the body would occur somewhere beyond or at the point of maximum load, when $K_s > 0$ or $K_s = 0$, in the diagram of F versus u_b (see Fig 1). Accordingly, the precursors, like damage localization and sensitivity of energy release, may commonly result from cascade of damage coalescence at various scales when approaching to the catastrophe transition point.

Damage Localization

We have introduced the condition for damage localization as [6-7]

$$\left(\frac{\partial\left(\frac{\partial D}{\partial y}\right)}{\partial t}\right) / \left(\frac{\partial D}{\partial y}\right) \geq \left(\frac{\partial D}{\partial t}\right) / D. \qquad (3)$$

Under the approximation of one dimensional quasi-static and small deformation, a lower bound for damage localization is

$$f_D \geq f /D . \qquad (4)$$

where f is the dynamic function of damage and can be expressed as a binary function $f(\sigma, D)$, and $f_D = \dfrac{\partial f}{\partial D}$. Clearly, if the dynamic function of damage is concave, there is almost always a tendency to damage localization beyond the point defined by Eq.(4). Actually, Eq.(4) implies enhancing cascade of damage coalescence in mean field approximation.

Critical Sensitivity of energy release

An earthquake may be viewed as a catastrophe transition to main rupture in heterogeneous crust. The cascade of damage coalescence is the underlying mechanism and causes informative fluctuations ahead of the transition. Accordingly, we examine the other measure of variation of the energy release rate --- sensitivity to external load. The sensitivity is defined as

$$S = \frac{\Delta E'/\Delta\sigma'}{\Delta E/\Delta\sigma} \qquad (5)$$

where ΔE and $\Delta E'$ are release energy induced by increments of stress $\Delta\sigma$ and $\Delta\sigma'$ respectively. When $S \sim 1$, this means that minor variation in governing parameters would not trigger any exaggerated consequences, or say, there are independent random events only in the heterogeneous region. Whilst, when $S > 1$ and becomes increasing, the system becomes sensitive and minor variation in governing parameters must induce multi-scale coalescence. Xia et al [8, 9] have shown that the sensitivity increases significantly prior to the catastrophe transition from a global stable (GS) stage to main rupture in evolution induced catastrophe (EIC) in a one-dimensional non-linear model. This feature is called critical sensitivity. Actually, the critical sensitivity is essentially rooted in the non-linear dynamics of the transition from global stable accumulation to catastrophic rupture.

Numerical Results

We performed numerical simulation to examine the rupture precursors based on the two concepts: damage localization and critical sensitivity.

The simulation was performed on a two-dimensional network model. The network model is made of a triangular, elastic truss and developed in [10]. All bars in the truss have the same elastic modulus K_0. However, their breaking strengths are different following a prescribed distribution. In the simulation, the distribution is assumed as a Weibull distribution function with two parameters: scale parameter η and Weibull modulus m (shape parameter). In the followings, we adopted the normalized stress σ = stress/η and normalized strain ε = strain * K_0 / η. So, the Weibull distribution

of the normalized strength of bars σ_c is

$$w(\sigma_c) = m \cdot \sigma_c^{m-1} \cdot \exp(-\sigma_c^m) \tag{6}$$

The result of mean field approximation of the above-mentioned heterogeneous model is shown in Fig.2. The Figure clearly demonstrates that damage localization (Eq(4)), maximum stress(corresponding to K_s=0 in Eq(2)) and catastrophic rupture(corresponding to K_s/K_0=1 in Eq(2)) occur successively with increasing deformation. An example of the simulated process of deformation and damage pattern in the heterogeneous model is demonstrated in Fig. 3. The prediction of damage localization made by mean field approximation (the cross in Fig 3) does provide a proper alarm. More importantly, the damage localization alarm is more sensitive than any other clues of rupture. For example, one cannot see any hints of rupture in the damage pattern at this stage, but damage localization does occur just a bit later. The sensitivity (Eq.(5)) of the same simulation is shown in Fig.4. The clear increase of the sensitivity prior to maximum stress provides the other alarm prior to rupture. This is in agreement with the results obtained in the one-dimensional model[9]. Hence, damage localization and sensitivity are two cross checking precursors of rupture.

Discussion

Both damage localization and sensitivity of energy release result from the cascade of micro-damage coalescence. However, damage localization is its representation in spatial evolution, whereas the sensitivity of energy release is its counterpart in the temporal sequence. The former can be examined in terms of a smoothly fitted dynamic function of damage or mean field approximation, whilst the latter can be calculated directly in the light of discrete intermediate-sized events. From the simulations, it can be seen that both damage localization and critical sensitivity of energy release can provide alarm prior to rupture. But, for accurate prediction of rupture, there is still a need to look closely at the relationship between various precursors and the main rupture.

Acknowledgements

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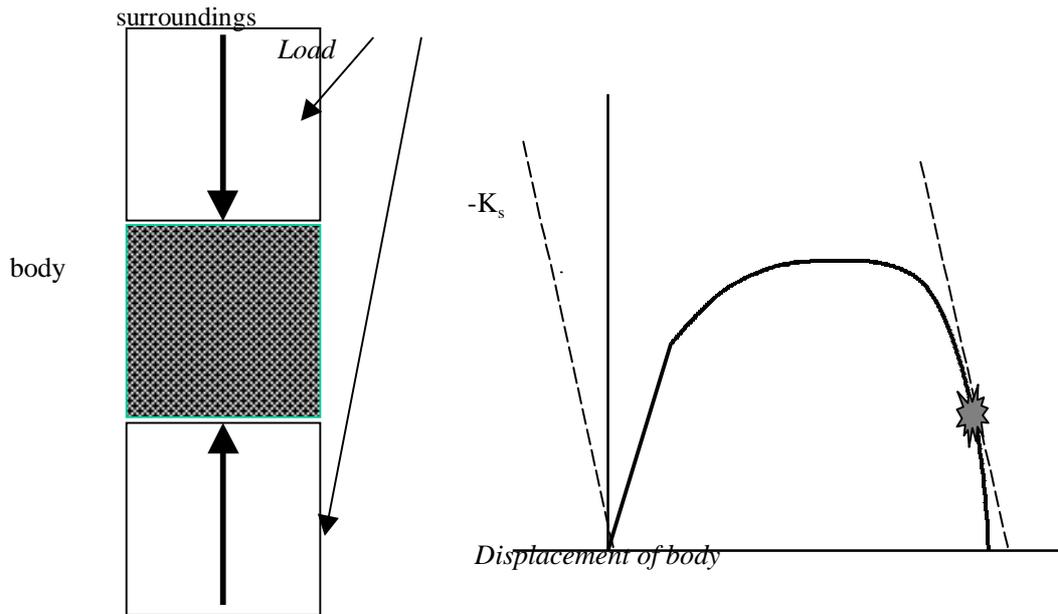
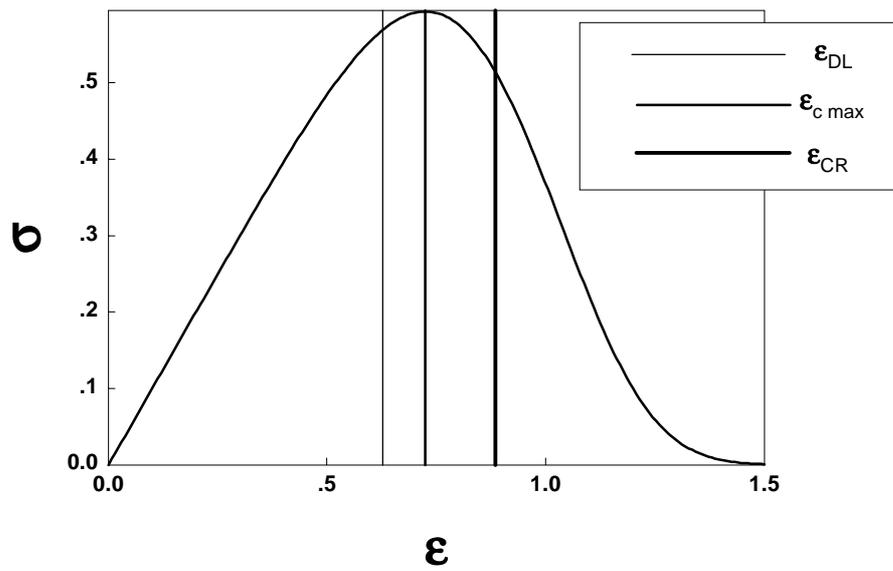Figure 1   Schematic of the catastrophe transition point of the body (denoted by ✹)

Figure 2 The stress – strain relation in mean field approximation of a heterogeneous model possessing Weibull distribution function with m=5. The three vertical lines (from left to right) indicate damage localization(ε_{DL}), maximum stress($\varepsilon_{c\,\text{max}}$) and catastrophic rupture(ε_{CR}) (when K_s/K_0=1) successively.
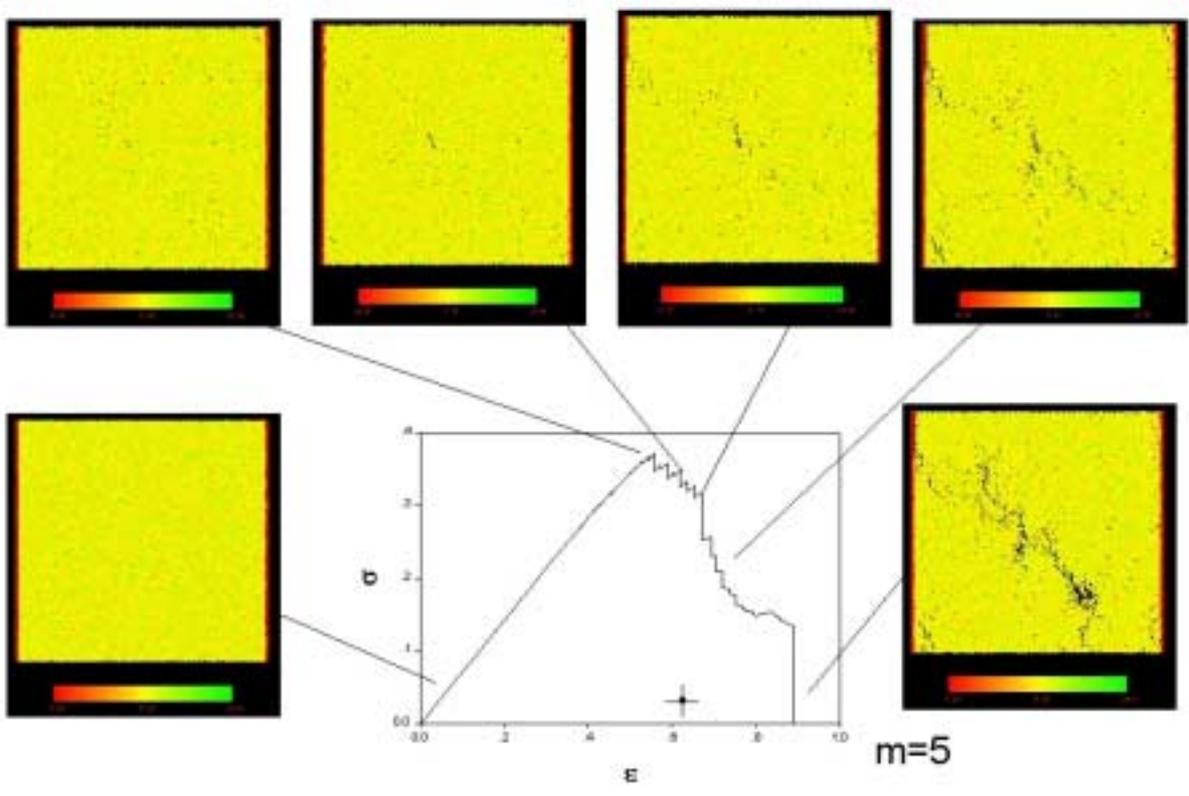

Figure 3   The simulated stress – strain relation of a heterogeneous model possessing Weibull distribution function with m=5 and corresponding damage patterns. The cross (+) indicates the damage localization condition (EQ(2)).
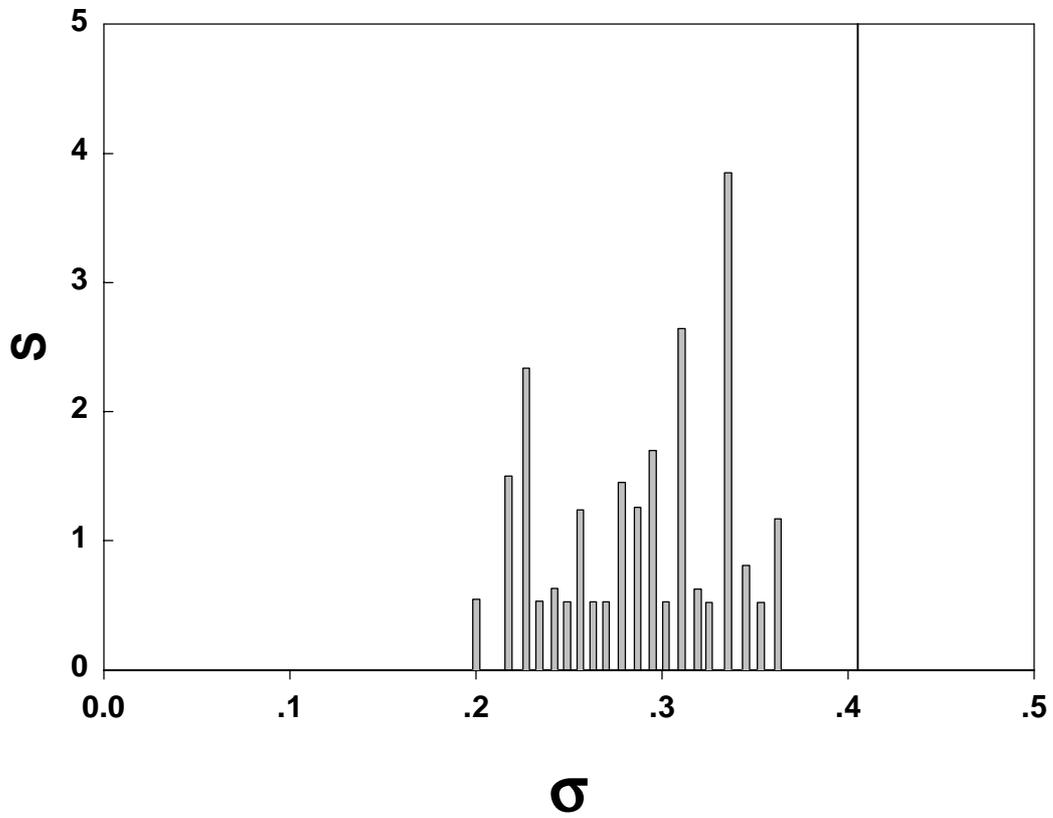
Figure 4   The sensitivity of energy release versus strain in a heterogeneous model possessing Weibull distribution function with m=5. The right vertical line indicates the position where maximum stress appears. Notice the rapid increase of the sensitivity when approaching the maximum stress.