

Scaling and Phase Transitions in Models of Earthquake Faults

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For many years it has been known that scaling is an integral part of earthquake phenomenology. Examples such as the Gutenberg-Richter law of magnitude frequency scaling, the Omori law for aftershocks and the Buffey-Varnes law for Benioff strain demonstrate that scaling is an integral part of the physics of the earthquake process. The natural question is; What is the physical process or processes that lead to these scaling laws which take the form of power laws. As an example; the number of earthquakes with magnitude M scales as $N(M) \sim M^{-b}$ where b is between 1.5 and 2.

If we examine other systems that produce scaling of this form; fluids magnets, glasses, certain biological systems or systems approaching chaos or turbulence, we generally find that the underlying physical processes are associated with critical points. At such points there is a balance between opposing tendencies. For example the tendency to maximize entropy and minimize entropy compete with each other in systems connected to heat baths. This leads to large fluctuations or areas of high correlation which produce scaling.

It is certainly not clear how critical points can arise in earthquake faults. To understand this point (among others) models such as the one created by Burridge and Knopoff(1967) were constructed. The difficulty with such models is that although they are extremely simple compared to earthquake faults they are still extremely complicated with respect to the underlying physics.

Our approach, which I will attempt to give you an overview of in this presentation is to begin with the simplest model that contains the essential physics of an earthquake fault. We identify the process or processes that are responsible for the scaling, and then begin to embellish the model with those characteristics which have been left out and examine their affect on scaling as well as other phenomena.

The key to this development is the realization that the stress Green's function can be seen from linear elasticity theory to vary as r^{-3} where r is the distance between blocks in the BK model or deformations on a strike slip fault. Since the second moment of this interaction is infinite in two dimensions the interaction is long range. The consequence of the long range interaction are substantial. Before we discuss them we need to point out that the interaction we use is long range but not infinite. That is, the second moment is large but not infinite. The physical reason is that that we believe that dirt, micro cracks, water etc will relax on a time scale that will "shield" the r^{-3} linear elastic term.

With this understanding we return briefly to the infinite range limit. In this limit the system becomes meanfield as shown in simpler systems by Kac, Lebowitz, Penrose et al. One of the results that we have found by investigating a simple cellular automaton version of the BK model first introduced by Rundle, Jackson and Brown(RJB) is that, in contrary to nearest neighbor models, the infinite range model(mean field) is described by an equilibrium theory. This means that in the infinite range limit there is a spinodal. This is a line of critical points which marks the limit of mechanical stability of the system. For long but finite range interactions the spinodal is not quite there but it is close to physical space. It is in fact in the complex driver plate velocity V and complex "temperature" space where the "temperature" is the amplitude of the noise used in the model. We will call this a pseudo-spinodal.

This nearby pseudo-spinodal influences the phenomena on "small" length scales that is on scales up to the correlation length. This length is large but finite. On larger length scales the system "senses" that the spinodal is not really there and that the equilibrium picture has broken down. Hence this picture has produced two predictions. First: There are two classes of events,

those that scale and are smaller than the correlation length and those that are larger and do not scale. Second; The scaling laws can be predicted by understanding the properties of the spinodal. Note that we are, at this stage, not concerned with anisotropic faults and finite size effects.

In addition, the model fault goes through a loading and unloading cycle. The fault model approaches the spinodal as it loads and goes away as earthquakes drain stress from the fault. This leads to scaling laws for after shocks and Benioff strain that are consistent with the Omari and Buffy-Varnes laws. The critical exponents then seem to be associated with the equilibrium part of the model spectrum. Larger events do not scale and cannot be treated with equilibrium methods.

We are presently studying these larger events and are also varying the model to see how different physical attributes affect the scaling.