Microscopic simulation of stress correlation evolution: implication for the Critical Point Hypothesis for earthquakes

Peter Mora[(1)] and David Place[(1)]

(1) QUAKES, Department of Earth Sciences, The University of Queensland, Brisbane, Australia (e-mail: mora@earth.uq.edu.au; place@earth.uq.edu.au, phone +61-7 3365 2128; +61-7 3365 9773, fax: +61-7 3365 7347).

Abstract

It has been argued that power-law time-to-failure fits for cumulative Benioff strain and an evolution in size-frequency statistics in the lead-up to large earthquakes is evidence that the crust behaves as a Critical Point (CP) system. If so, intermediate–term earthquake prediction is possible. However, this hypothesis has not been proven. If the crust does behave as a CP system, stress correlation lengths should grow in the lead-up to large events and drop sharply once these occur. However, this evolution in stress correlation lengths cannot be observed directly. Here, we show using the lattice solid model to describe discontinuous elasto-dynamic systems subjected to shear and compression, that it is for possible correlation lengths to exhibit CP-type evolution. This provides the first direct evidence that systems with realistic elasto-dynamics may behave as CP systems, and hence supports the Critical Point hypothesis for earthquakes.

Introduction

The power law size-frequency statistics of earthquakes has been cited as evidence that the earth's crust is in a self-organised critical state, and hence, that earthquake prediction is impossible (Geller et al., 1997[4]). On the other hand, accelerating Benioff strain release (Bufe and Varnes, 1993[3]; Bowman et al, 1998[2]) and an evolution in earthquake statistics prior to large events (Jaumé, 2000[5]) is cited as evidence that the crust is not perpetually in a critical state, but acts as a critical-point system in which there is an approach and retreats from criticality. In this view, an earthquake cycle is proposed to proceed as follows. Initially, small earthquakes occur and redistribute stress locally. This process gradually allows long-range stress correlations to be established which are argued to be a pre-condition for a rupture to runaway to the largest event size (Rundle et. al, 1999[14]). Once this occurs, the large earthquake will destroy the long-range stress correlations and the cycle will repeat. But this evolution in stress correlations is impossible to observe directly in the earth. Numerical simulations provide a means to study this possibility.

The lattice solid model

The lattice solid model (LSM) is a particle-based numerical model that was developed to simulate the nonlinear dynamics of earthquakes (Mora and Place, 1992[6],1993[7],1994[8]; Place and Mora, 1999[12]). It is presently capable of simulating physical processes such as friction, fracture, granular dynamics and thermal effects including thermo-mechanical and thermo-porous feedback (Abe et al., 2000[1]). The LSM has been applied to the study of the heat-flow paradox (Mora and Place, 1998[9]; 1999[10]), rock fracture and localisation phenomena (Place & Mora, 2000[13]). Recently, shear experiments involving a granular system exhibited power-law acceleration in Benioff strain release and evolving frequency-magnitude statistics in the lead-up to large events

(Mora et al. 2000[11]) suggesting the lattice solid model may be used to probe the Critical Point question.

Shear experiments

Numerical experiments were conducted using a granular model subjected to shear. Since the model allows ruptures to occur along any internal surface within the granular region, the numerical model can be considered as a simplified model for an interacting fault system. Figure 1 shows the deviatoric stress (σ_1 - σ_2) in the central granular region which was sheared from the outer edges of elastic blocks above and below this region. Boundary conditions were circular in the *x*-direction. The deviatoric stress shows a complex pattern and has a filamentary appearance that reflects paths of high stress where potential future ruptures may occur. Typically, large events involve stress dropping suddenly on such paths and being redistributed elsewhere.

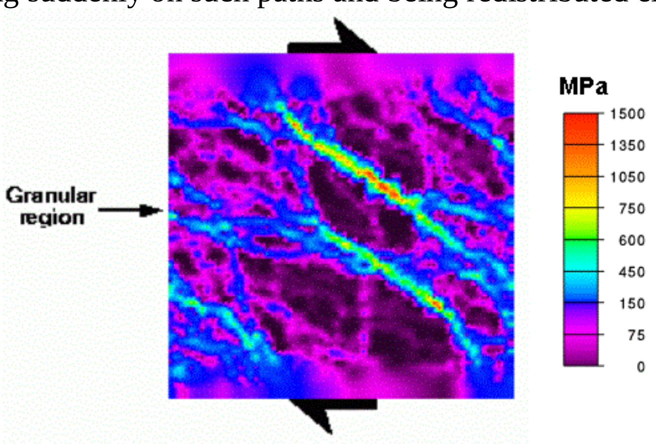

Figure 1: Deviatoric stress in the central granular region of a shear experiment. A normal stress of σ_{zz}150 MPa is maintained on the upper and lower edges of the elastic blocks above and below this region.

In the numerical experiment (see also, Mora et al, 2000[11]), the large events are often preceded by a period of accelerating Benioff strain release and an increased rate of moderate to large events (Figure 2). Best fit lines and power law time-to-failure functions were computed for the cumulative Benioff strain where the power law function (Sornette and Sammis, 1995[15]) had the form $\varepsilon(t) = A + B\,(t\text{-}t_f)^c\,[1 + D\cos(2\pi \log(t_f\text{-}t)/\log(\lambda) + \psi)]$. In the example, the best-fit power law exponent was c=0.38. The RMS error of the power law fit divided by the RMS error of the linear fit was 0.56 and 0.42 respectively for the cases without and with log-periodic fluctuations.

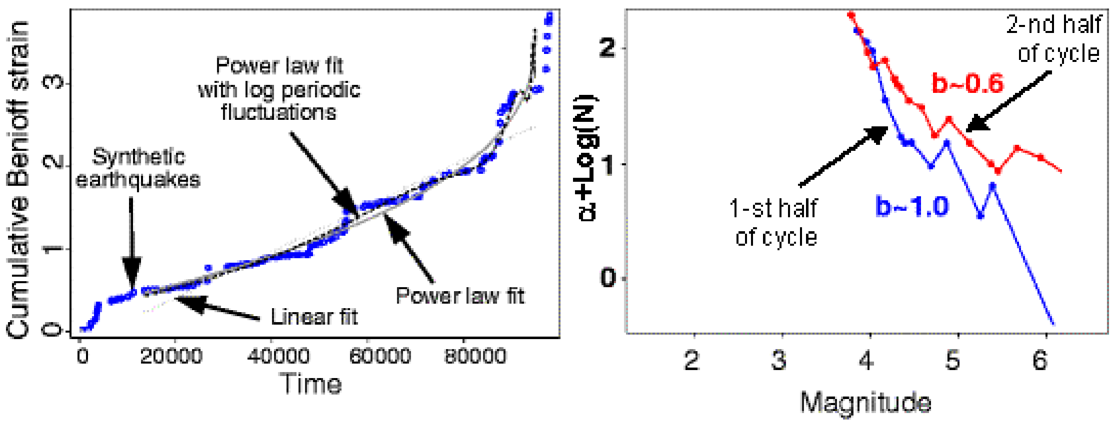

Figure 2: **Left:** Cumulative Benioff strain release for a simulated earthquake cycle along with linear and power law fits (dotted=linear, solid=power law, dashed=power law with log-periodic fluctuations). **Right:** Interval frequency-magnitude plot where α=constant and total number of earthquakes is 227.

For this sequence, we calculated the stress correlation function $C(r) = <\Delta\sigma(\mathbf{x})\Delta\sigma(\mathbf{x}+\mathbf{r})> / N$ where N is a normalization factor such that $C_{\max} = 1$ and $\Delta\sigma = \sigma - \bar{\sigma}$ with $\sigma = \sigma_1 - \sigma_2$ denoting the deviatoric stress, $\bar{\sigma}$ the mean deviatoric stress, and σ_1 and σ_2 the two principle stresses. The averaging is taken over all positions **x** and **x+r** within the granular region, and all vectors **r** of length $r = |\mathbf{r}|$. Figure 3 shows the evolution of the stress correlation function and demonstrates that correlation lengths progressively grow throughout the sequence and sharply drop when the largest event at the end of the sequence occurs.

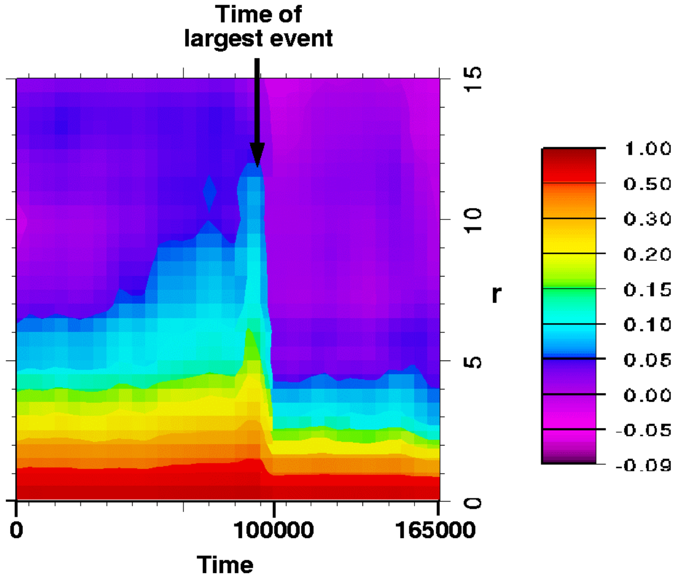

Figure 3: Evolution of the stress correlation function with time for the shear experiment.

Compression experiment

The shear experiment presented above involved a pre-existing granular layer. In the following, we initialize a model with random particles bonded by elastic brittle bonds and subject it to uniaxial compression in the *y*-direction. Snapshots of the simulation are shown in Figure 4 and illustrate that after about Time stepx100 = 1000, a well-developed fracture system has developed.

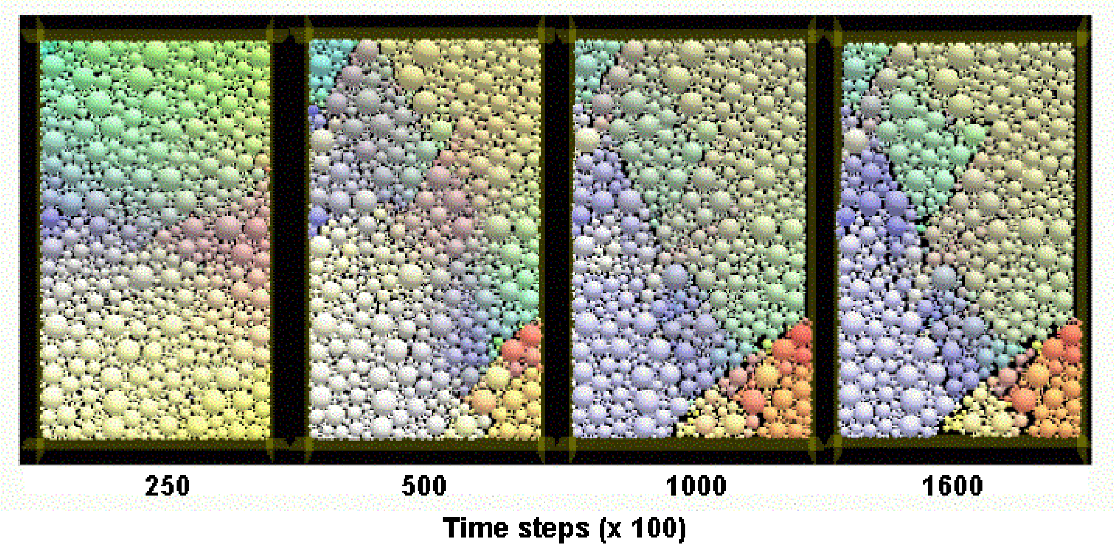

Figure 4: Snapshots in a compression experiment.

A plot of stress as a function of time is shown in Figure 5. Since the driving rate on the upper and lower edges was constant, this plot depicts the stress-strain curve. Sharp drops in stress indicate rupture events. Early in the simulation, these correspond to development of new fractures but later the rupture events may also involve slip along the existing fracture surfaces.

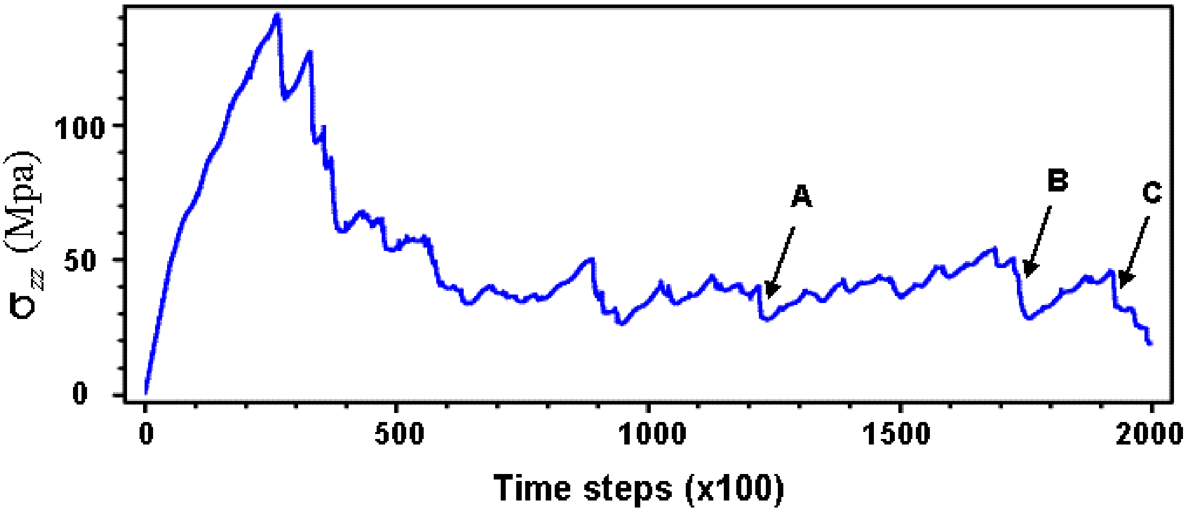

Figure 5: Stress versus time = stress-strain curve for the uniaxial compression experiment.

Figure 6 shows the evolution of the stress correlation function with time. One observes that early in the simulation (Time step x 100 <1000), there is no clear evolution effect like that seen in the shear experiment. However, later in the simulation there are several examples where the stress correlation function appears to evolve in a manner consistent with the CP hypothesis. Namely, prior to the large events labeled A, B and C when stress drops sharply on Figure 5, the correlation length grows followed by a sharp drop when a large event occurs.

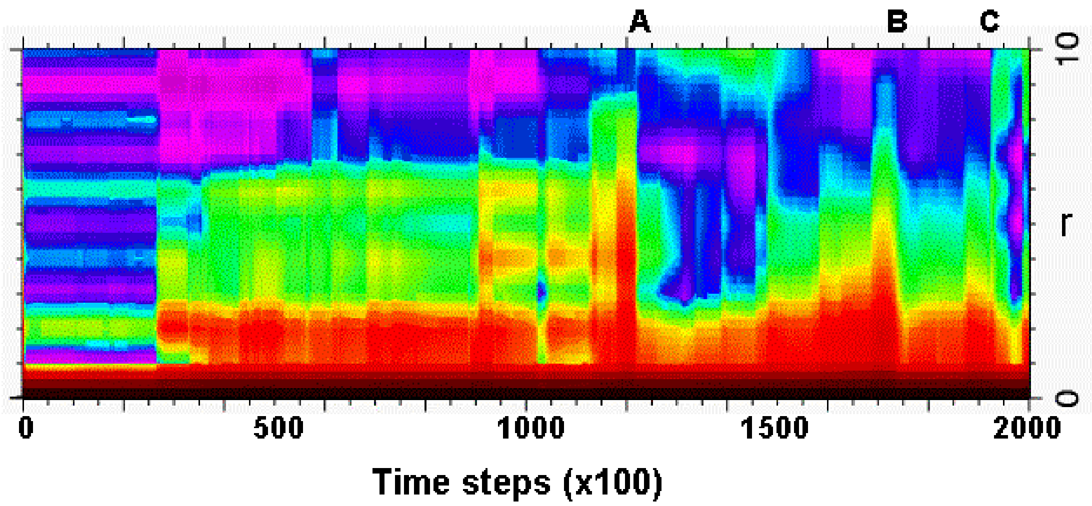

Figure 6: Evolution of the stress correlation function with time for the uniaxial compression experiment using the same color scale is in Figure 3.

Discussion

The shear experiment sequence showed an evolution in the stress correlation function consistent with the Critical Point Hypothesis for earthquakes. In the case of the compression experiment, this evolution became evident only after establishment of a well-developed fracture system. The results suggest that well developed systems of interacting faults may behave as CP systems but that regions with few interacting faults will not. If so, earthquake forecasting will rely on our ability to characterize the system dynamics of any given region.

Collaborative research between Australia and China using the lattice solid model aiming to study system behaviour, critical sensitivity and the underlying mechanism for observations of accelerating Benioff strain release and the Load-Unload Response Ratio has commenced (Weatherley et al, 2001[17]; Wang et al, 2001[16]; Yin et. al, 2001[18])

Conclusion

The numerical experiments prove that it is possible for an elasto-dynamic system to exhibit an evolution in stress correlations consistent with the Critical Point Hypothesis for earthquakes. The results therefore suggest that intermediate-term earthquake forecasting is achievable, at least for certain fault systems.

Acknowledgments

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