

Elastodynamic analysis of earthquake sequences on slowly loaded faults with rate and state friction

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Abstract

Lapusta et al., 2000[2] have developed an efficient and rigorous numerical procedure for elastodynamic analysis of earthquake sequences on slowly loaded faults. This is done for a general class of rate and state friction laws with positive direct velocity effect. We use the procedure to study the response of a 2-D strike-slip fault model with depth-variable properties. We find the following (as partially reported by Lapusta et al., 2000[2]): Small events appear in increasing numbers for decreasing values of the characteristic slip distance of the friction law. The nucleation phase of small and large events is very similar. For a large event that is preceded by a small event (and hence heterogeneous stress distribution), moment acceleration in the beginning of dynamic propagation exhibits "slow-downs" and subsequent "speed-ups", consistently with some observations. Insufficient time and space discretization qualitatively changes the results. Incorporating slow tectonic loading is essential for uncovering the true model response.

Introduction

Elastodynamic analysis of earthquake sequences on slowly loaded faults is a very challenging problem. It requires simulating thousands of years of slow tectonic loading and potential slow aseismic slip while resolving wave propagation effects for each model earthquake (which lasts seconds or minutes). Various solutions have been proposed by other researchers: To employ a quasi-static method during slow deformation and then to switch (abruptly) to a dynamic method once an instability starts; To neglect all aseismic fault slippage, so that stressing between earthquakes is trivially modeled; To impose a plate loading rate which is many orders of magnitude larger than for natural faults and to use standard elastodynamic numerical methodology throughout (Lapusta et al., 2000[2] and references therein).

Lapusta et al., 2000[2] have developed an integrated numerical scheme allowing resolution of both slow and fast deformational phases, as well as of the transition between them, within a single mathematical framework for elastodynamics. The method enables us to perform calculations over thousands of years of slow tectonic loading, punctuated by earthquakes and the processes which lead to and follow them. Thus we can resolve aseismic slip on velocity-strengthening fault regions, advance of slip into more firmly locked zones, and slowly accelerating aseismic slippage that grows in spatial extent and will ultimately break out into an earthquake but has duration that is vastly longer than the seismic event itself. We also resolve all details of the break out of rupture, its propagation and arrest, and the transient post seismic slippage that develops.

Methodology: Two main ingredients

Truncation of dynamic response

The first ingredient is based on the form of elastodynamic relations that we use, in which the dependence of the inertial response on prior deformation history can be truncated so that only a (fixed) part of the deformation history back from current time needs to be considered. That translates into fixed memory requirements and fixed amount of computation per each time step. It also makes the computation at each time step independent of how much time has already been simulated. We use spectral representation of elastodynamic relations in which the slip distribution is represented as a Fourier series in the spatial coordinate, truncated at large order, and fast Fourier transform (FFT) methods are used. Our algorithm can also be generalized to the (closely related) space-time boundary integral formulation.

Variable time stepping

The second ingredient is variable time stepping. The size of the time step to be made is dictated by the current values of slip velocities and parameters of the constitutive law. The smaller the slip velocities, the larger the time step, and vice versa. Throughout the computation, time steps can change by many orders of magnitude in value, allowing us to go in relatively few steps through periods of essentially quasi-static loading, to consider more carefully the nucleation phase, and to resolve in great detail the features of the dynamic propagation during an instability. The coefficients of proportionality between the time steps and the inverse of slip velocities depend on the parameters of the constitutive law as well as on numerical stability considerations that we have derived. We have developed the formulation for a general class of rate- and state-dependent friction laws with a positive direct velocity effect. The presence and size of the positive direct velocity effect for the quasi-static range of slip velocities, amply documented in experiments, are crucial in allowing long time steps during slow deformation phases without losing stability.

Implementation example

We apply the procedure to a 2-D vertical strike-slip fault in which slip is constrained to vary with depth only (there is no variation along strike). The fault is loaded with the equivalent of plate velocity of 35 mm/yr. We model the free surface effects using a mirror image. The problem can be expressed in the 2-D anti-plane framework. The constitutive law used is a regularized version of the Dieterich-Ruina (logarithmic) rate and state friction. The fault has depth-variable frictional properties. It exhibits steady-state velocity strengthening friction right next to the free surface and at the bottom, while the region in between has steady-state velocity-weakening properties and hence can produce model earthquakes. The detailed description of the model is given in Lapusta et al., 2000[2].

More small events for smaller values of characteristic slip distance

Rate and state friction incorporates a characteristic slip distance L for evolution of frictional strength. For fixed other parameters, the nucleation size (size of the quasi-statically slipping patch that precedes the dynamic rupture break-out) is proportional to L . We study how earthquake sequences change as we decrease L , approaching the laboratory range of tens of microns (which are too small to be computationally feasible). As L decreases, small

events appear near the brittle-ductile transition at the bottom of the seismogenic (velocity-weakening) zone. For the particular depth-variable fault model studied by Lapusta et al., 2000 [2], a simulation with $L = 8$ mm produces a periodic sequence of large events, while a simulation with $L = 2$ mm results in the sequence of a large and a small event (Figure 1), as do the simulations with $L = 1$ mm and 0.5 mm. In the case with $L = 0.14$ mm (done so far only quasi-dynamically; we intend to redo it with full dynamics), the sequence is more elaborate, with large events interspersed by three smaller events (Figure 2).

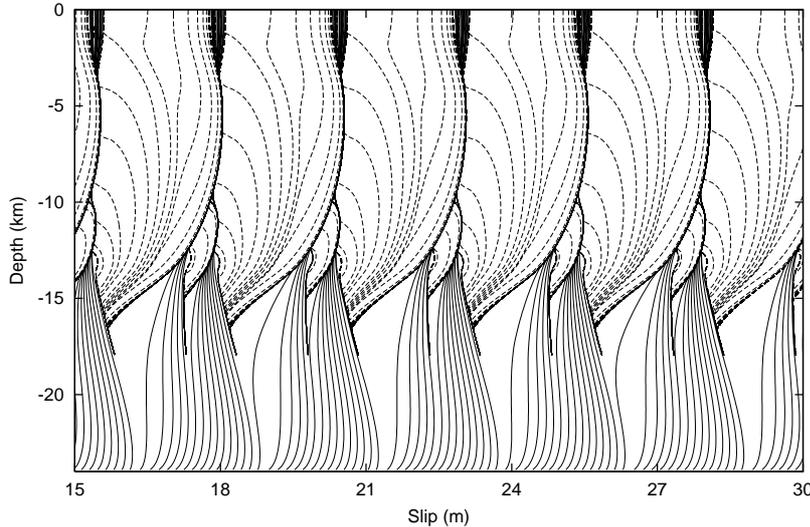


Figure 1: Accumulation of slip versus depth for the case $L = 2$ mm. The solid lines are plotted every 5 years. The dashed lines are plotted above 18 km depth every second if the maximum velocity anywhere on the fault exceeds 0.001 m/s. Small events appear at the bottom of the seismogenic zone. (From Lapusta et al., 2000[2].)

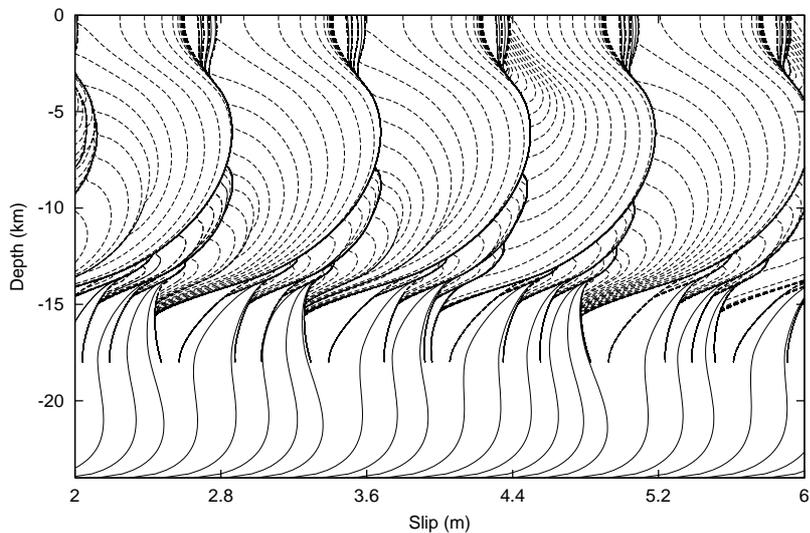


Figure 2: Accumulation of slip versus depth for the case $L = 0.14$ mm. Each large event is followed by a small event, then an "intermediate" event, and then another small event. Comparing to the case $L = 2$ mm in Figure 1, we see that smaller L results in more small events.

Similarity of nucleation process for small and large events

By "nucleation process" we mean the quasi-static slip (i.e., slip with negligible inertial effects) in a small expanding zone before the break-out of the dynamic event. We observe this nucleation in our simulations (Figure 3). The transition to inertial slip (with high, seismic slip velocities) is accompanied by the rapid expansion of the slipping zone, with rupture velocities that are a significant fraction of the shear wave speed. In our definition, this

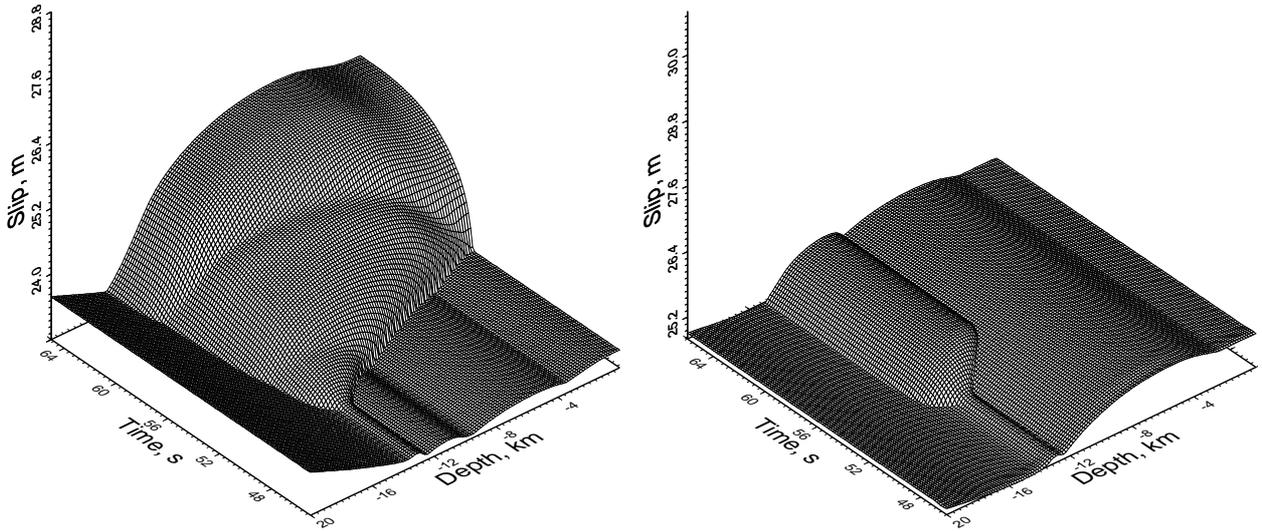


Figure 3: Slip in a large event (left) and the following small event (right) from the sequence in Figure 1. Zero in time is chosen arbitrarily for the plotting convenience. Notice the nucleation zone which actually extends long back in time. The nucleation process and the beginning of the dynamic break-out is very similar for the large and small events. The small event arrests by not being able to advance into the region of larger previous slip. (From Lapusta et al., 2000[2].)

expansion signals that the nucleation phase has ended and dynamic rupture propagation has begun. The nucleation of the large and small events is very similar, as manifested by plots of slip (Figure 3), slip velocity, moment rate, and moment acceleration. This means that observing the nucleation and beginning of a model earthquake, it is impossible to tell whether the final size of the event will be large or small. The final size of the event is determined by the conditions on the fault region that the event is propagating into, rather than by the nucleation process.

Irregular moment release of large events as affected by prior small events

We observe that moment acceleration during initial stages of dynamic rupture propagation can have "slow-downs" and subsequent "speed-ups". This is consistent with observations, e.g. as reported by Ellsworth and Beroza, 1995[1], who attribute these features to either processes

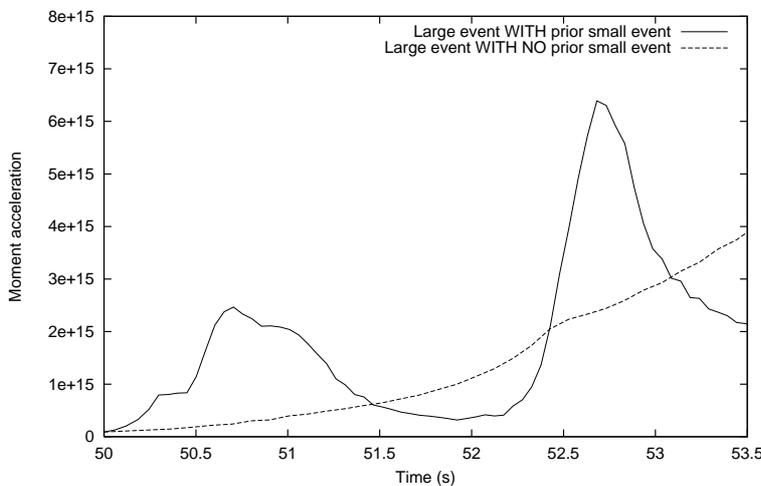


Figure 4: Moment acceleration during the initial stages of dynamic rupture propagation for a large event preceded by a small event (solid line) and for a large event that is not preceded by a small event (dashed line). Velocity seismograms in the far field are proportional to the moment acceleration. The large event that is preceded by a small event (and hence by heterogeneous stress due to the slip and arrest of the small event) has irregular moment acceleration, similar to observations.

in the preslip region or a special cascade structure of the fault. Our simulations show that such irregular moment acceleration (to which velocity seismograms are proportional) can be caused by heterogeneous stress distribution left by previous events. In the case of $L = 2$ mm, a large event is preceded by a small event (Figure 1). For such a large event, moment acceleration grows initially, then decreases almost to zero during rupture propagation over the region that slipped in the previous small event, and then again abruptly grows, even faster than initially, when the rupture reaches the region of stress concentration left by the arrest of the small event (Figure 4, solid line). Such "slow-down" and "speed-up" are not observed for a large event in the case of $L = 8$ mm that does not have small events (Figure 4, dashed line).

Proper time and space discretization

Insufficient time and space discretization tends to produce artificial complexification of the earthquake sequences. Hence it is very important to verify the independence of the results on numerics by establishing convergence of the results (or at least of their qualitative features) as the parameters of the simulation are refined. This is often necessary even when the output looks smooth and plausible, as it can still be qualitatively different from the true response of the model. Usually, the plots of slip velocity or stress reveal much more about the numerical stability and convergence than their slip counterparts.

Importance of incorporating slow tectonic loading

The developed methodology, which incorporates both truly slow tectonic loading and all dynamic effects, can be used to evaluate simplified approaches. We have considered two such approaches: (I) a procedure with truly slow tectonic loading but with a part of the dynamic effects (namely, dynamic stress transfers) ignored (as in Rice, 1993[3]) and (II) a procedure with all dynamic effects incorporated but much faster loading (as in Shaw and Rice, 2000[4]). We find that, in the particular model considered, elimination of dynamic stress transfers slightly decreases slip per event and significantly decreases slip and rupture velocities, but produces no overall qualitative difference in the event sequence. The much faster loading, however, totally alters the nucleation process and location and results in significantly smaller slip per event. Hence simulating slow tectonic loading is very important for uncovering the true model response. The results also demonstrate that even though simplified approaches may be unavoidable in some cases, they have limitations that can be uncovered and remedied only within a more general approach like the one presented here.

Conclusion and future goals

The developed methodology has a number of significant advantages. It is capable of rigorous treatment of long-duration deformation histories with continuing aseismic creep slippage in velocity-strengthening fault regions throughout the loading period, with gradual nucleation of model earthquakes followed by dynamic propagation of ruptures, and with rapid post seismic deformation after such events. The model earthquakes produced have realistic features such as the well-resolved nucleation phase, seismic slip velocities of the order of meters per second, and rupture velocities of the order of kilometers per second. However, the methodology in its present form is not immediately applicable to problems that lack translational invariance (i.e., a fault oblique to the free surface or a layered Earth structure).

The short-term goals are to continue to use the methodology in a 2-D context to address a number of important issues, such as further studies of the earthquake nucleation process, fault operation under low overall stress, interaction of dynamic rupture propagation with pore pressure development, patterns of rupture propagation in events nucleated naturally as a part of a sequence, earthquake sequences on faults with heterogeneous frictional properties and/or normal stress, and others.

In the long term, two main goals are important. The first one is to extend this rigorous treatment (which incorporates slow tectonic loading, all dynamic effects, and experimentally derived friction laws) to a 3-D setting. This is straightforward conceptually, but presents a significant computational challenge. Our experience with 2-D problems suggests that, in the near future, 3-D simulations with realistic frictional parameters and rigorous numerical resolution are hardly feasible, and many problems would have to be studied in a 2-D context. The second goal is to extend the ideas of the methodology to cases lacking translational invariance. This could potentially be done by using finite difference or finite element procedures, not in their conventional application to directly calculate the rupture propagation itself, but rather to calculate and numerically tabulate the convolution kernels (for use in our methodology).

Acknowledgments

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