On development of crustal deformation/stress state monitoring system

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Abstract

This paper presents the stress inversion method which has been applied to the nationwide GPS network. This method determines a distribution of stress increment satisfying the equilibrium from a distribution of measured strain increment (Hori et al., 2000[1]). We are now developing a system of monitoring crust deformation and stress state using the stress inversion method. The system will be also used in constructing a reliable model of the Japanese Islands which account for the regional heterogeneity.

Introduction

The nationwide GPS network is most advanced technology to monitor the curst deformation of the Japanese Islands. Monitoring stress state is needed as well for the earthquake prediction. To this end, authors are proposing the application of the stress inversion method to predict the stress increment associated with the strain increment measured by the network (Hori *et al.*, 2000[1]). Monitoring stress and strain also contributes the development of reliable physical model of Japan; see Fig. 1.

The stress inversion regards the Japanese Islands as a thin plate at plane stress state during a period of the GPS measurement. The input data are the strain increment distribution measured by the GPS network, and in-plane stress increment is predicted such that they satisfy the equilibrium equations. The inversion method needs traction along the boundary, which can be predicted by considering the plate movement. This means that the following two are assumed as sources for the stress increment: 1) the regional heterogeneity that induces the local and particular change in strain increment; and 2) the plate movement that causes the overall and consistent deformation of the Japanese Islands.

In this paper, we present the stress inversion method and a proptotype of a crustal deformation/stress state monitoring system. The basic formulation and some preliminary results of analyzing the GPS data are shown. The inversion for predicting the plate movement is briefly presented.

Formulation

Stress inversion method

The key idea of the stress inversion method is simple, as it takes advantage of the two equilibrium equations for three in-plane stress components. It is more straightforward to use Airy's stress function, a, instead of the stress components, σ_{ij}, i.e.,

$$[\sigma_{11}, \sigma_{22}, \sigma_{12}]^T(\mathbf{x}) = [a_{,22}, a_{,11}, -a_{,12}]^T(\mathbf{x}), \tag{1}$$

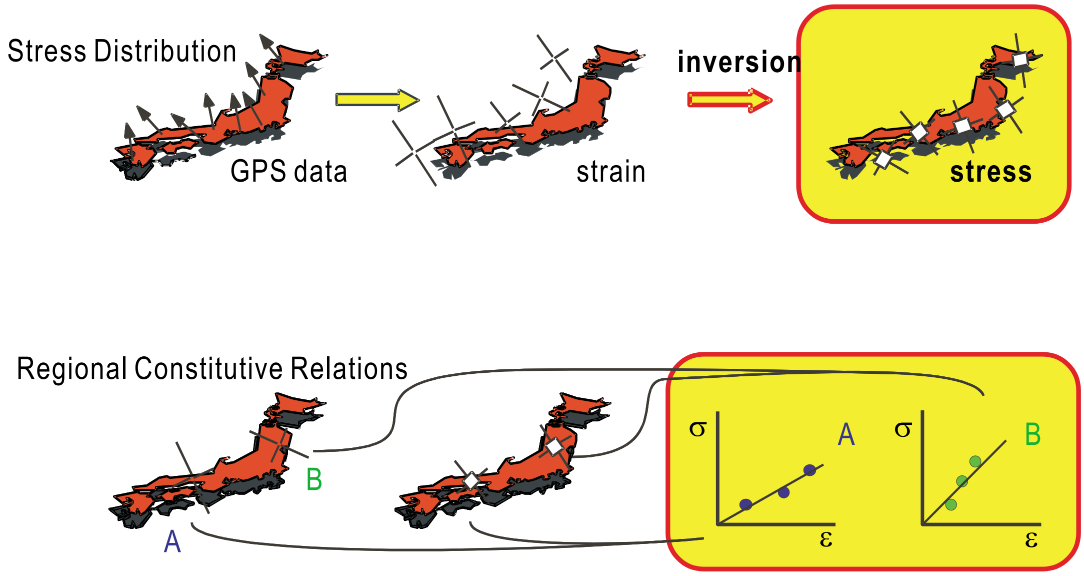


Figure 1: Usage of GPS network for monitoring stress increment and modeling Japanese Islands.

where $,i$ stands for $\partial/\partial x_i$. The stress components in the right side of Eq. (1) automatically satisfy the equilibrium. Making an assumption that the volumetric stress increment, $\dot{\sigma}_{11}+\dot{\sigma}_{22}$, is linearly related to the volumetric strain increment, $\dot{\epsilon}_{11} + \dot{\epsilon}_{22}$, we can derive the governing equation for a, as

$$\dot{a}_{,11}(\mathbf{x}) + \dot{a}_{,22}(\mathbf{x}) = \kappa(\dot{\epsilon}_{11}(\mathbf{x}) + \dot{\epsilon}_{22}(\mathbf{x})). \quad (2)$$

Here, the strain increment in the left side is given by the GPS network and κ is assumed to be constant. The boundary conditions of Eq. (2) are found if traction increment along the boundary is known as $\dot{t}_i$,

$$n_1(\mathbf{x})\dot{a}_{,1}(\mathbf{x}) + n_2(\mathbf{x})\dot{a}_{,2}(\mathbf{x}) = n_1(\mathbf{x})\dot{r}_1(\mathbf{x}) + n_2(\mathbf{x})\dot{r}_2(\mathbf{x}), \quad (3)$$

where n_i is the unit norm and $\dot{r}_i$ is the resultant traction increment, $\dot{r}_i = \int \dot{t}_i \, \mathrm{d}\ell$. As is seen, Eqs. (2) and (3) form a boundary value problem for $\dot{a}$.

It should be emphasized that the validity of the assumption, $\dot{\sigma}_{ii} = \kappa\dot{\epsilon}_{ii}$, is not verified even though it means that major regional heterogeneity is due to sliding of active faults. We need to examine relations between the measured strain increment and the predicted stress increment to verify the validity. Finding κ may be required. It should be also mentioned that a similar boundary value problem is formulated if another assumption is made; for instance, if the isotropy is assumed, we have

$$(\dot{a}_{11} - \dot{a}_{22})\dot{a}_{,12} + \dot{\epsilon}_{12}(\dot{a}_{,11} - \dot{a}_{,22}) = 0.$$

Spectral decomposition of Green function

Since the boundary value problem of $\dot{a}$ is linear, we can decompose $\dot{a}$ into two parts, the one determined by the surface strain increment and the other determined by the boundary traction increment. The second part corresponds to the plate movement, and we assume that it is interpreted as the back slip distribution along the plate boundary. We are now

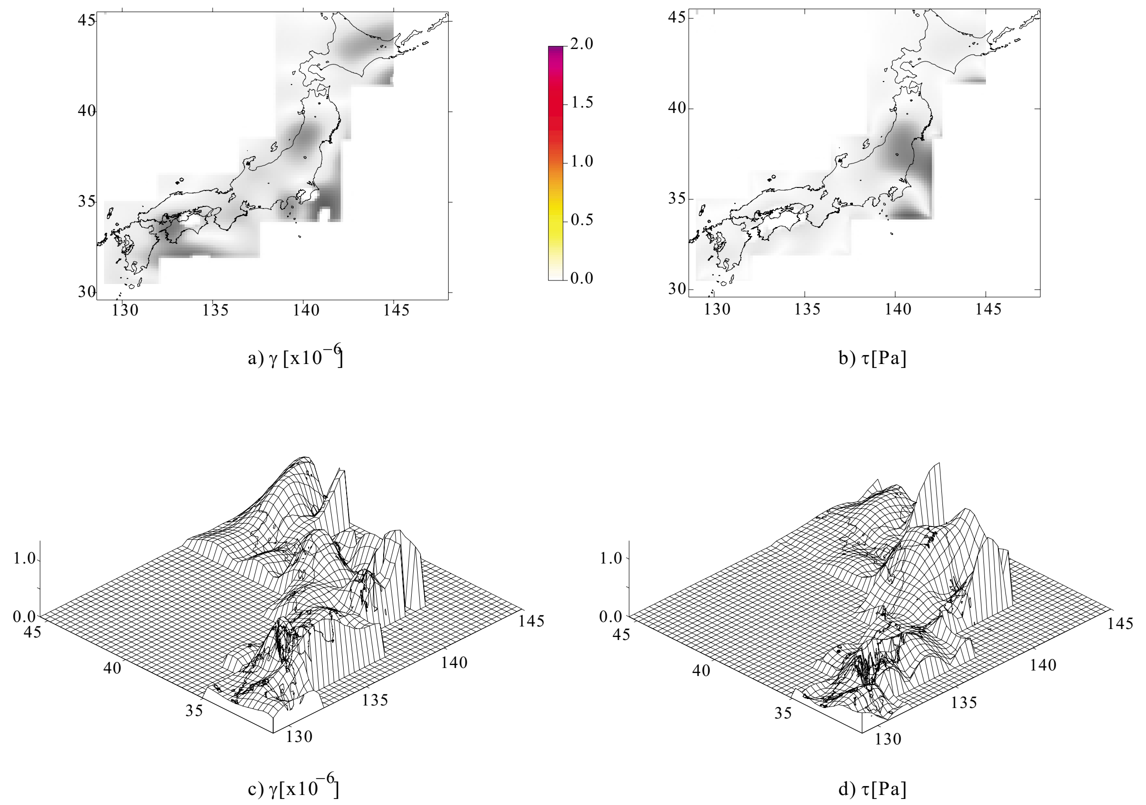


Figure 2: Example of stress inversion method: κ corresponding to Young's modulus of 1Pa.

developing a new method which is based on the spectral decomposition of Green's function, such that the displacement increment, not the strain increment, is used for the inversion (Hori, 2000[2]).

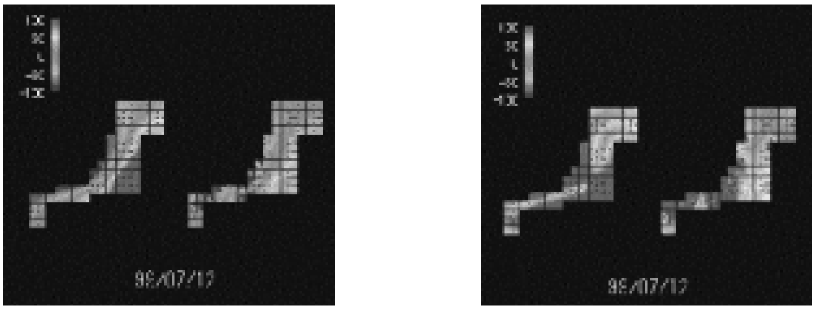
The spectral decomposition of Green's function is to find a few dominant modes of displacement distribution. To simplify the notation, we consider Green's function g which relates a source p on a fault plane F to a response u measured on a part of the surface S, i.e., $u(\mathbf{x}) = \int_F \; g(\mathbf{x}, \mathbf{y})p(\mathbf{y}) \; \mathrm{d}\mathbf{y}$. Then, g is decomposed as

$$g(\mathbf{x}, \mathbf{y}) = \sum_\alpha \lambda^\alpha \phi^\alpha(\mathbf{x}) \psi^\alpha(\mathbf{y}) \quad \left(\int_S \; \phi^\alpha \phi^\beta \; \mathrm{d}\mathbf{x} = \int_F \; \psi^\alpha \psi^\beta \; \mathrm{d}\mathbf{y} = \delta^{\alpha\beta} \right). \tag{4}$$

Here, λ^α vanishes as α increases. Thus, only a few modes of ϕ^α appear on S, and u is given as $\sum u^\alpha \phi^\alpha$. Hence, the inversion first determines a few ua's using measured data and then predicts the source as $p = \sum (u^\alpha / \lambda^\alpha) \psi^\alpha$. For instance, denoting by u^n the measured value of the response at the n-th measuring point $\mathbf{x}^n$, we can determine u^α's by

$$\text{minimize} \; \sum_n \left(u^n - \sum u^\alpha \phi^\alpha(\mathbf{x}^n) \right)^2.$$

Finding $\{\lambda^\alpha, \phi^\alpha, \psi^\alpha\}$ in Eq. (4) is straightforward even though it requires a large amount of computation.



a) first invariant b) second invariant

Figure 3: Example of crustal deformation/stress state monitoring: right and left are for measured strain and predicted stress (values are normalized as the maximum becomes 100).

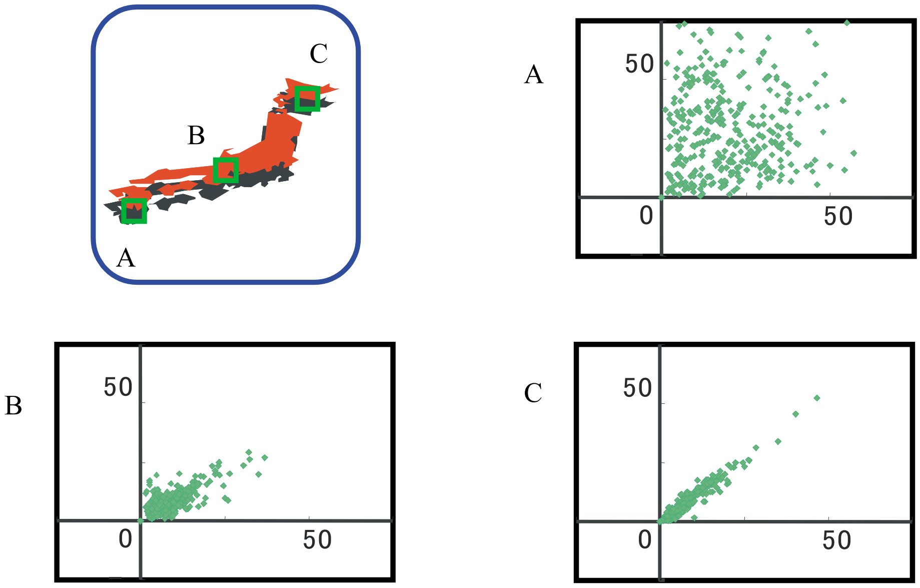


Figure 4: Examples of regional stress-strain relation.

Results of computation

Monitoring of regional deformation/stress state

Some preliminary checks have been made in order to examine the applicability of the stress inversion method to the GPS data (Hori *et al.*, 2000[1]); see Fig. 2 for the distribution of the second invariant distribution of the measured strain and the predicted stress. They are the convergence of the numerical solution, the effects of the boundary conditions and the choice of κ. The data are strain increment between 1997 and 1998, and the distribution filtered by the least squares prediction is used. The results support the basic applicability of the stress inversion method.

As a prototype of the crustal deformation/stress state monitoring, we develop a set of codes which analyze GPS data measured every day. The codes determine the strain increments using the least squares prediction, and predict the stress increment applying the stress inversion method. Figure 3 shows an example of the analysis; a) and b) are the first

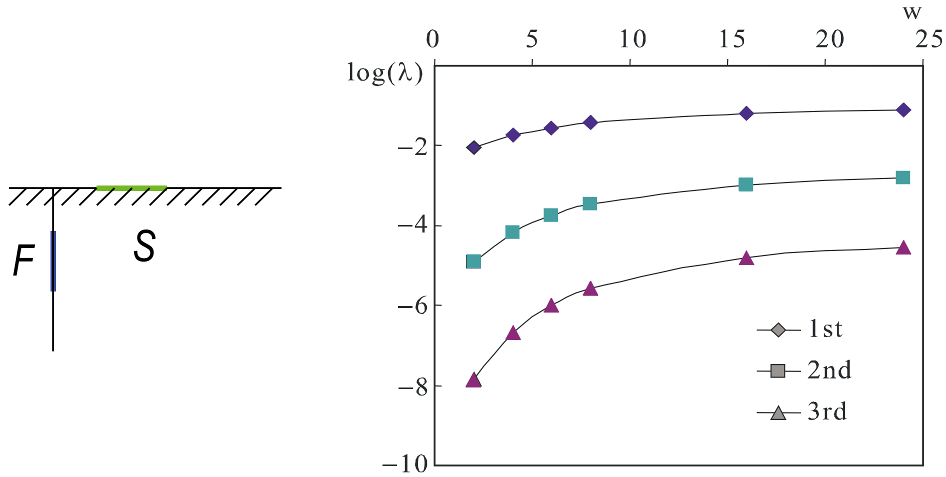


Figure 5: Change in first three λ^α's with respect to size of measurement region.

and second invariant for the measured strain increment and the predicted stress increment, respectively. It should be emphasized that the daily strain increment may have significant noises. Hence, the results of using such data may be meaningless. The prototype, however, can be used if such noises are omitted or if monthly increment are used as input.

In Fig. 4, we plot the second invariant of the stress increment and the strain increment at three regions. Some linear relations can be seen for the regions B and C. The validity of these relations is questioned, since the input data may have large noise. These figures just demonstrate a possible usage of the crustal deformation/stress state monitoring system.

Example of spectral decomposition

Since Green's function decays rapidly, we can well expect that λ^α in Eq. (4) vanishes fast. Vanishing of λ^α is more serious when a target body is a semi-infinite body and measurement is made only on the surface. To clarify this point, we carried out a simple numerical simulation of a two-dimensional homogeneous half plane, using Green's function of a potential problem, $g(x,y) = \frac{-1}{\pi}\,\frac{x}{x^2+y^2}$, which corresponds to a double couple of an elastic body (Hori, 2000[2]).

Figure 5 plots the change of $\lambda^{1,2,3}$ as a wider region is used for the measurement. As expected, the values increase, which means that the accuracy of the inversion increases. However, λ^2 and λ^3 are smaller by orders than λ^1. When the accuracy of the measurement is limited, we may not be able to find the second mode corresponding to λ^2.

Concluding remarks

We are expecting that the stress inversion method could be used to develop a crustal deformation/stress state monitoring system which makes use of data measured by the nationwide GPS network. The prototype has been developed, even thought the validity of the analysis is not examined at all at this momenet.

There are several major difficulties in developing a reliable monitoring system. For instance, we need to examine the assumption made in the stress inversion method. Also, we implement the inversion method based on the spectral decomposition of Green's function to predict boundary traction increment.

Acknowledgments

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