Macro-micro analysis method for large-scale computation of wave propagation processes

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Abstract

This paper presents the macro-micro analysis for the prediction of the strong motion distribution in a metropolis (Ichimura and Hori, 2000[1] and Hori and Ichimura, 2000[2]). The key features of the method are the stochastic modeling of the crust and ground structure and the multi-scale analysis for higher spatial and time resolution. The formulation of the macro-micro analysis is presented. An actual earthquake is simulated, and the results are compared with measured data, showing the basic of validity of the macro-micro analysis.

Introduction

The prediction of the strong motion that is caused by a huge earthquake is a key issue for earthquake engineering. A more accurate estimate is needed for dynamic analysis of structure or for the micro zoning of earthquake hazards. Since the strong motion is the final phase of an earthquake, the prediction must account for the propagation through crust and the amplification near ground surfaces as well as the initiation at a source fault.

There are two major difficulties in numerically simulating the propagation and amplification processes. The first difficulty is the uncertainty of the crust and ground structures. Due to the limitation of quality and quantity of the measurement, the underground structures are not fully determined. The second difficulty is the computational complexity that increases drastically as higher accuracy and resolution are needed. The length scale required for engineering purposes is of the order of 1m, which leads to a huge amount of discretization in computing.

Recently, the authors have been proposing a new analysis method to predict the strong motion distribution in a metropolis. This method is called *macro-micro analysis*, and has the following two key features:

1. **bounding medium theory:** As an alternative of a deterministic model, a stochastic model is used for the underground of a metropolis. The bounding medium theory determines two fictitious but deterministic bodies for the stochastic model, such that their responses provide both optimistic and pessimistic estimates of the mean behavior of the stochastic model.

2. **singular perturbation expansion**: The singular perturbation expansion is applied to a displacement field to reduce computational efforts by carrying out a multi-scale analysis. The displacement is given as the sum of the first order solution with lower spatial resolution for the whole city and the second order solution of higher spatial resolution for each small part of the city.

Figure 1 presents a schematic view of the macro-micro analysis.

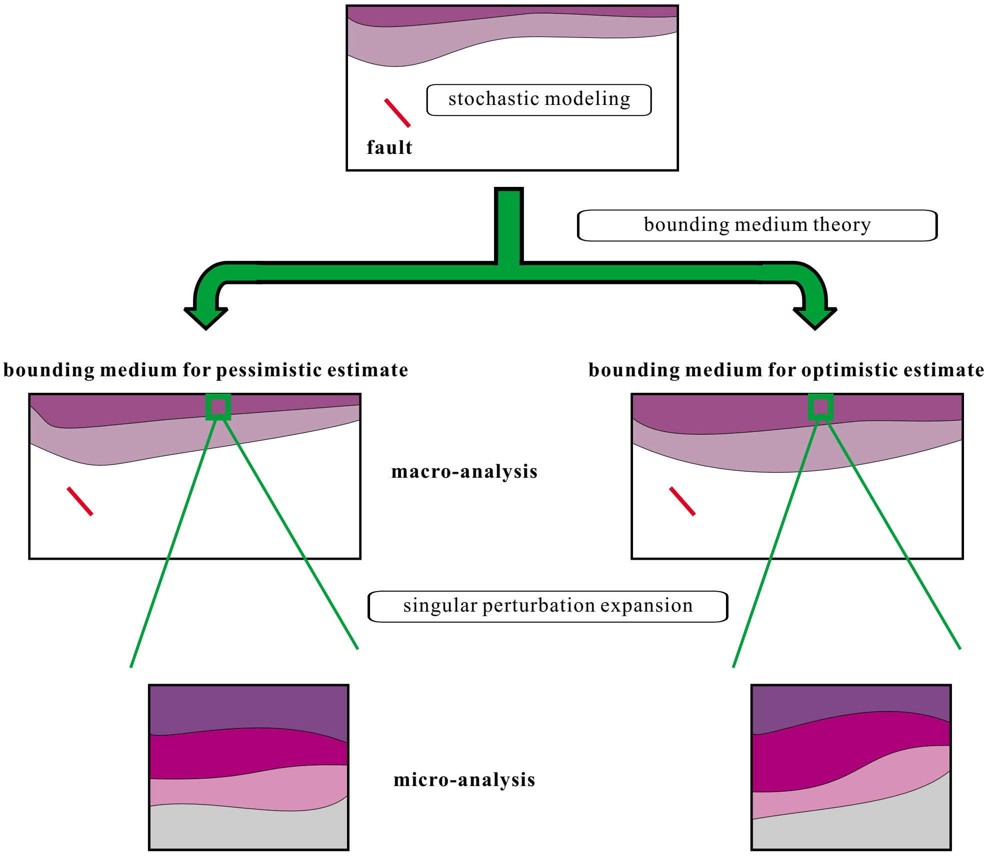


Figure 1: Key features of macro-micro analysis.

Formulation

The bounding medium theory is general as it is applicable to various physical problems which are described in a variational form. As the simplest example, we consider a stochastic model for a linear elastic body V with varying elasticity c_{ijkl} at quasi-static state. One bounding medium is determined as

$$c^{+}_{ijkl}(\mathbf{x}) = \langle \mathbf{c_{ijkl}} \rangle(\mathbf{x}), \tag{1}$$

where $\langle\ \rangle$ stands for the stochastic mean. This is because the following inequality holds for the mean of the total strain energy, denoted by $\langle E \rangle$, of the stochastic model:

$$\langle E \rangle = \langle J(\mathbf{u^e}, \mathbf{c}) \rangle < \langle \mathbf{J}(\mathbf{u}, \mathbf{c}) \rangle = \mathbf{J}(\mathbf{u}, \langle \mathbf{c} \rangle).$$

Here, when c_{ijkl} is realized for the stochastic model, J is the potential energy ($J = \int_V \frac{1}{2} c_{ijkl} \mathrm{d}_j u_i \mathrm{d}_l u_k \, \mathrm{d}V$ with $d_i = \partial/\partial x_i$) and u^e_i is the displacement field. Another bounding medium is found by considering the complementary potential energy, as

$$c^{-}_{ijkl}(\mathbf{x}) = \langle (\mathbf{c})^{-1}_{\mathbf{ijkl}} \rangle^{-1}(\mathbf{x}). \tag{2}$$

It should be emphasized that $c^{\pm}_{ijkl}$ given by Eqs. (1) and (2) provide bound for $\langle E \rangle$ of the quasi-static state. We use these media for the dynamic state, assuming that the inertia effects are not dominant.

Once the bounding media are given, we take singular perturbation expansion for displacement, introducing a *slow* variable $X_i = \varepsilon x_i$ with ε, which gives the ratio of the ground structure length scale and the crust structure length scale. Denoting by c_{ijkl} and ρ the elasticity and the density of the bounding medium, we arrive at $u_I \approx u_i^{(0)}(\mathbf{X},\mathbf{t}) + \varepsilon \mathbf{u_I^{(1)}}(\mathbf{X},\mathbf{x},\mathbf{t})$, and the first and second terms satisfy

$$D_i(C_{ijkl}D_l u_k^{(0)}) - R\ddot{u}_j^{(0)} = 0, \tag{3}$$

$$d_i(c_{ijkl}(\mathrm{d}_l u_k^{(1)} + \mathrm{D}_l u_k^{(0)})) - \rho(\ddot{u}_j^{(1)} = 0, \tag{4}$$

where c_{ijkl} is either c_{ijkl}^{+} or c_{ijkl}^{-}, C_{ijkl} and R are the *effective* elasticity and density given by c_{ijkl} and ρ, and D_i and d_i are $D_i = \partial/\partial X_i$ and $d_i = \partial/\partial x_i$, respectively. In numerical computation, Eq. (4) is reduced to

$$d_i(c_{ijkl}(\mathrm{d}_l u_k^{(1)} + \mathrm{d}_l u_k^{(0)})) - \rho(\ddot{u}_j^{(1)} + \ddot{u}_j^{(0)}) = 0, \tag{5}$$

by changing the argument of $u_i^{(0)}$ and dropping ε for $u_i^{(1)}$. The analysis of computing $u_i^{(0)}$ and $u_i^{(1)}$ are called the macro-analysis and the micro-analysis, respectively.

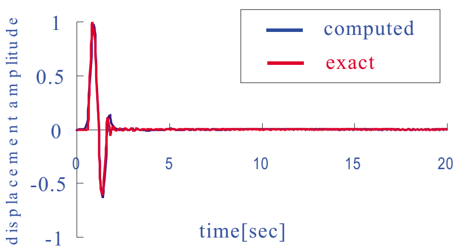
In numerically solving Eq. (3) and Eq. (5), we apply the voxel finite element method (VFEM) to reduce required memory storage. The VFEM discretizes the medium using identical cubic elements which have a common element stiffness matrix, and hence increases the computational efficiency using an iterative solver. We are also studying the boundary element method implemented by the fast multi-pole method to solve Eq. (3), which could be most efficient in solving a three-dimensional dynamic problem.

Results of computation

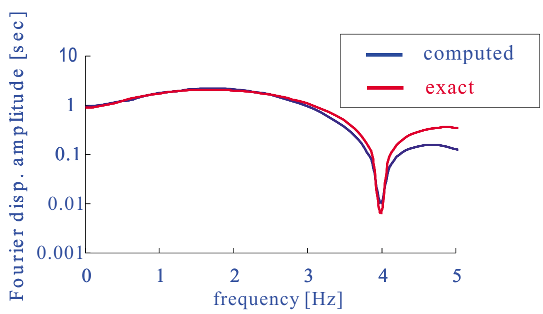
Validity of VFEM for dynamic problem

First, we examine the validity of the VFEM to solve dynamic problems. Three-dimensional wave propagation caused by an explosion source is solved. The key settings of the simulation are the Wilson θ method for the time integration and the paraxial boundary conditions for artificial boundaries. The element dimension is 240×240×240m and the P wave velocities are 5200m/s.

The numerical results are compared with the analytical solutions. The results are presented in Fig. 2. It is seen that the VFEM can compute the wave up to 2Hz, which means that a harmonic wave of wave length 2600m is computed by using the discretization of 240m.



a) time domain



b) frequency domain

Figure 2: Comparison of numerical solution and analytical solution.

Results of macro-micro analysis

Next, we carry out the macro-micro analysis for an actual earthquake; see Table 1. The data measured at six sites within the Yokohama City are used for the comparison; see Fig. 3 for the locations of sites and the bounding media for the macro-analysis and the micro-analysis.

Table 1: Properties of earthquake.

Lat.	Long.	Depth	Strike	Dip	Rake	Mag.
35.4N	149.8E	53km	62^{o}	85^{o}	73^{o}	4.0Mw

Figure 4 a) presents the comparison of the macro-analysis with the measured data; the optimistic bounding medium is used and velocity in the time domain is plotted. Since the time resolution is 1.2Hz, we filter higher frequencies in the figure. The agreement is satisfactory even though the simplest Haskell model is used for the fault mechanism. Therefore, the macro-analysis can simulate the wave propagation process to some extent of the accuracy and the resolution. However, the site effects on the wave amplification is not well simulated; see Fig. 5 for the velocity spectrum.

Figure 4 b) presents the comparison of the micro-analysis with the measured data. While the two bounding media do not bound the measured data, the wave profile is close to them. Figure 6 plots the velocity spectral distribution of the six site. As is seen, the site effects are simulated in the micro-analysis that modifies the macro-analysis by accounting for the local ground structures.

Concluding remarks

The results of the numerical simulation shown in the preceding section supports the basic validity of the macro-micro analysis. The two bounding media needs to be interpreted such that the results could be used for the practical purposes; for instance, the average and the difference of the two media may be understood as an approximation of the mean and the variance of the stochastic behavior.

In order to obtain the reliable prediction, we still need to increase the discretization used in the macro-micro analysis. Numerical techniques for the efficient parallel computation will be studied. Also, the macro-micro analysis will be implemented for the function of analyzing non-linear behavior of ground surfaces.

Acknowledgments

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References

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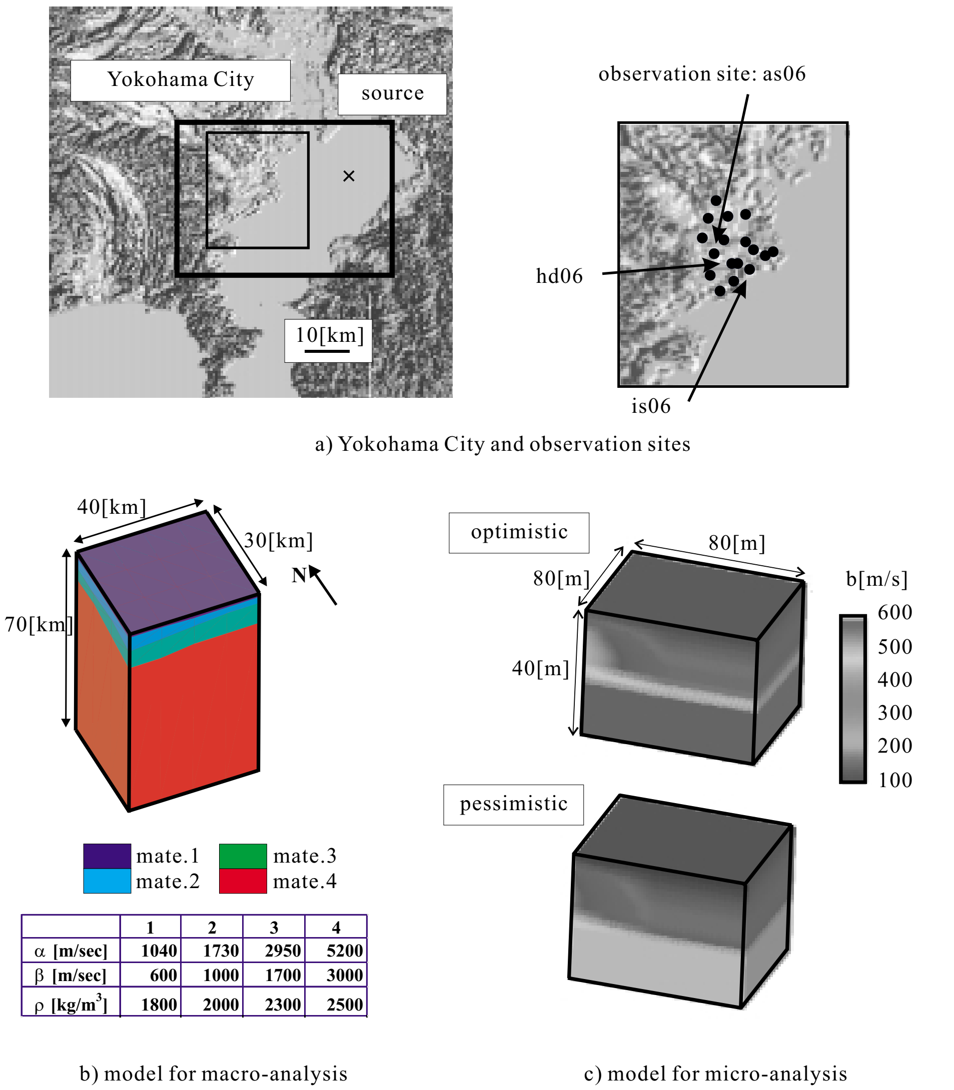


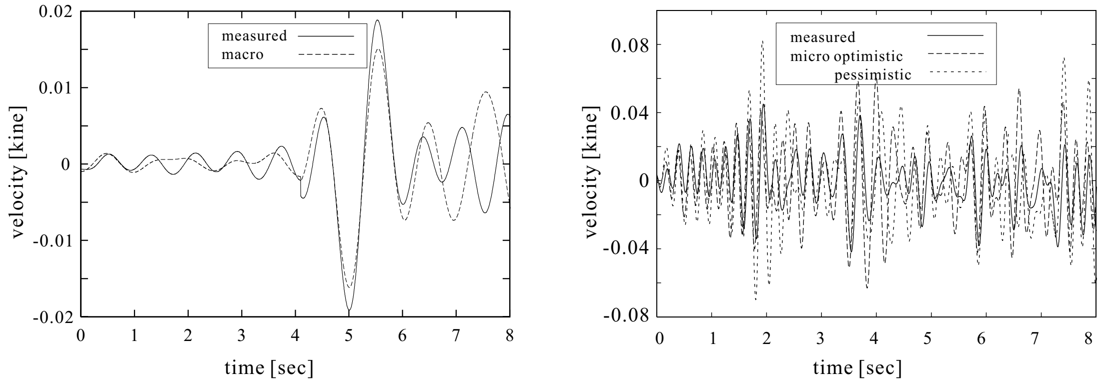
	1	2	3	4
α [m/sec]	1040	1730	2950	5200
β [m/sec]	600	1000	1700	3000
ρ [kg/m^3]	1800	2000	2300	2500

b) model for macro-analysis

c) model for micro-analysis

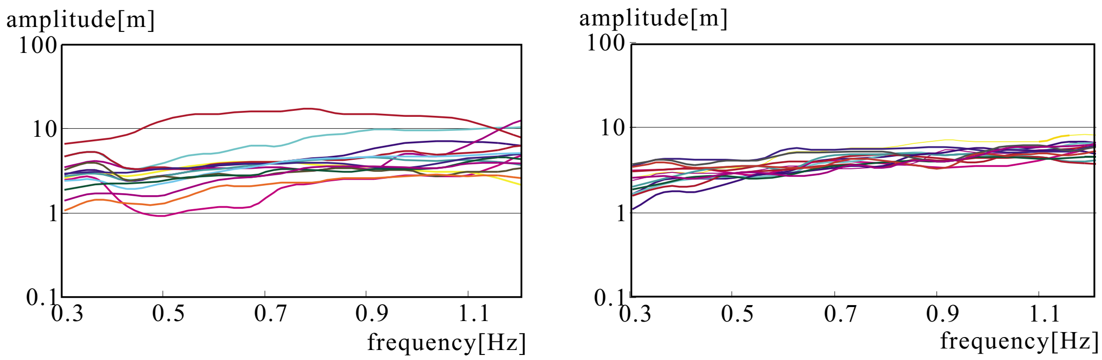
Figure 3: Model of macro-micro analysis; a) for the observation sites, b) for the macro-analysis model and c) for the micro-analysis model.

[2] Hori, M. and Ichimura, T., 2000 *Macro-micro analysis for wave propagation in highly heterogeneous media,* in International Workshop WAVE2000, Bochum, Germany, Decomeber 13-15, 2000 (to be printed).



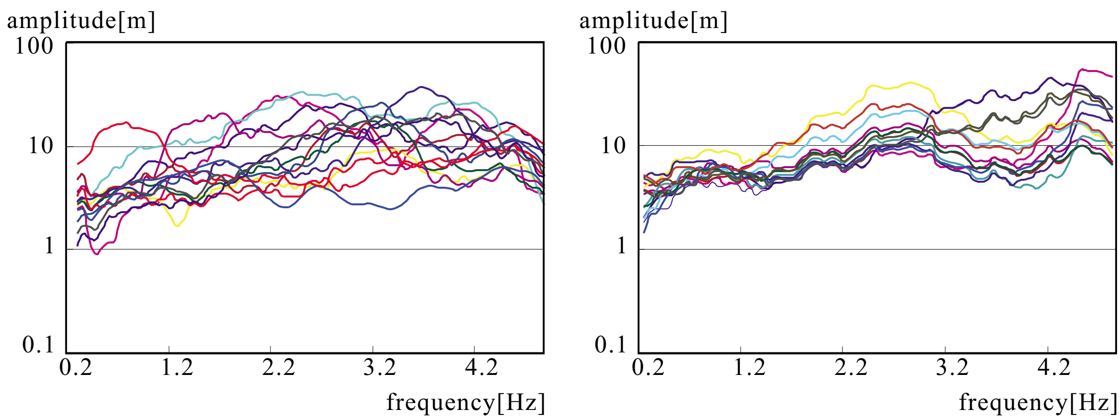
a) macro-analysis b) micro-analysis

Figure 4: Velocity at as06 obtained by macro-micro analysis.



a) measured b) macro-analysis

Figure 5: Velocity spectrum obtained by macro-analysis.



a) measured b) micro-analasis: pessimistic

Figure 6: Velocity spectrum obtained by micro-analysis.