A constitutive scaling law that unifies the shear rupture from small scale in the laboratory to large scale in the Earth as an earthquake source

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Abstract

In order to develop a realistic model for the earthquake nucleation to dynamic rupture with prediction capability, it is essential to incorporate the physical scale dependence into the model. This can be attained if geometric irregularity of the rupturing surfaces is properly incorporated into the model, and if the governing law for earthquake rupture is formulated as a slip-dependent constitutive law. It is shown that scale-dependent physical quantities, such as the breakdown zone size, the nucleation zone size, the apparent shear rupture energy, and the slip acceleration, inherent in the shear rupture in a broad scale range from 10^{-2} to 10^{5} m can be understood unifyingly and consistently in terms of a laboratory-based slip-dependent constitutive law in the framework of fracture mechanics.

Introduction

Rupture (or fracture) phenomena, including the earthquake rupture, are observed in a very broad range from an atomistic scale, through a microscopic scale, to a macroscopic scale. Laboratory-scale rupture, including the shear fracture of intact materials and the frictional slip failure on a pre-existing fault, may roughly be of the order of 10^{-3} to 1 m, whereas field-scale rupture, including micro- to large earthquakes, may be of the order of 10^{-1} to 10^{5} m. It is widely recognized that some of the physical quantities inherent in the rupture are scale-dependent. In order to develop a realistic, physical model for the earthquake nucleation to dynamic rupture with prediction capability, it is therefore essential to incorporate the physical scale dependence into the model. The theme of the physical scaling dependence has been addressed at the ACES inaugural workshop held in Brisbane and Noosa, Australia, and I emphasized there that the governing law (constitutive law) should be formulated so as to scale scale-dependent physical quantities inherent in the rupture, and thereby a unified comprehension should be provided for the shear rupture of any size scale which continuum mechanics encompasses – small scale in the laboratory to large scale in the Earth as an earthquake source. This can be attained for the shear rupture if geometric irregularity of the rupture surfaces is properly incorporated into the model, and if the governing law for the shear rupture is formulated as a slip-dependent constitutive law. These two are the key to scaling scale-dependent physical quantities inherent in the shear rupture. In this paper, it will be shown how unifyingly and consistently scale-dependent rupture phenomena in a broad scale range from 10^{-2} to 10^{5} m can be understood in terms of a laboratory-based constitutive law.

A constitutive scaling law

It has been established that the earthquake source at shallow crustal depths is a shear rupture instability that takes place on a fault characterized by *inhomogeneities* (e.g., Aki, 1984[1]). An inhomogeneous fault includes local, strong patches of high resistance to rupture growth. Such strong patches are needed for an adequate amount of the elastic strain energy to be stored in the surrounding medium, which is necessary as a driving force to bring about a large earthquake. Strong patches of high resistance to rupture growth are also needed for generating strong motion seismic waves. If a fault is very weak everywhere on the entire fault, little amount of the elastic strain energy will be stored in the surrounding medium, so that a large (or strong) earthquake cannot be generated. Thus, the presence of strong patches is required on the fault. In addition, there is a very important, physical constraint imposed on the patch strength; that is, the upper end member of the patch strength is the shear fracture strength of initially intact rock. Therefore, if there is a constitutive law that governs the earthquake rupture, the law should be formulated as a unifying constitutive law that governs both frictional slip failure and shear fracture of intact rock mass.

As discussed at the ACES inaugural workshop, the physical constraint mentioned above necessarily leads to the conclusion that the constitutive law for the earthquake rupture should be formulated as a slip-dependent constitutive law. This is because it is a slip-dependent constitutive law that governs the shear fracture process of initially intact rock. The slip-dependent constitutive law is a unifying law that governs both frictional slip failure and shear fracture of intact rock. Indeed, it has become increasingly clear that laboratory data on both frictional slip failure and shear fracture of intact rock are unified consistently by a single law of slip-dependent constitutive formulation. A slip-dependent constitutive relation is uniquely specified by the following five parameters: the initial strength τ_i on the verge of slip at the rupture front, the peak shear strength τ_p attained at the slip displacement D_a, the breakdown stress drop $\Delta\tau_b$ defined as the stress difference between τ_p and the residual frictional stress, and the breakdown slip displacement D_c defined as the critical amount of slip required for the shear strength to degrade to the residual frictional stress. The slip-dependent constitutive formulation presumes the slip displacement to be an independent and essential variable, and the rate- or time-dependence to be of secondary significance. Thus, the shear traction is expressed as a function of the slip displacement in this formulation, and the parameters prescribing the law, such as τ_i, τ_p, $\Delta\tau_b$, and D_c, are assumed to be an implicit function of the rate or time (Ohnaka et al., 1997[11]).

It has been found that the constitutive law parameters τ_p, $\Delta\tau_b$, and D_c (or equivalently $D_{wc} = D_c - D_a$) are interdependent, and that they are mutually constrained by the following relation:

$$\Delta\tau_b/\tau_p = \beta\left(D_c/\lambda_c\right)^M \tag{1}$$

or

$$\Delta\tau_b/\tau_p = \beta'\left(D_{wc}/\lambda_c\right)^M \tag{2}$$

where β, β' and M are numerical constants determined from laboratory experiments (β=1.6, β'=2.1, and M=1.2 for granite rock in the brittle regime), and λ_c denotes the characteristic length representing a predominant wavelength component contained in geometric irregularity (roughness) of the rupturing surfaces. It has also been found that laboratory data on both shear fracture of intact rock and frictional slip failure are unified consistently by a single relation (1) or equivalent relation (2).

Rewriting relation (1) leads to

$$D_c = m(\Delta\tau_b/\tau_p)\lambda_c \tag{3}$$

where $m(\Delta\tau_b/\tau_p)$ is a dimensionless parameter expressed as a function of $\Delta\tau_b/\tau_p$ as follows

$$m(\Delta\tau_b/\tau_p) = (1/\beta)^{1/M}\left(\Delta\tau_b/\tau_p\right)^{1/M} \tag{4}$$

Similarly we have

$$D_{wc} = \left(\beta/\beta'\right)^{1/M} m(\Delta\tau_b/\tau_p)\lambda_c \tag{5}$$

and

$$D_a = \left[1 - (\beta/\beta')^{1/M}\right] m(\Delta\tau_b/\tau_p)\lambda_c \quad (6)$$

Relations (3), (5) and (6) show that all the displacement parameters D_c, D_{wc}, and D_a scale with λ_c, since $\Delta\tau_b/\tau_p$ is scale-independent. The slip-dependent constitutive law includes these scale-dependent displacement parameters D_c, D_{wc}, and D_a, and hence the constitutive law for the shear rupture is inherently scale-dependent. This scale-dependent property of the constitutive law is essential for describing scale-dependent rupture phenomena in quantitative terms.

Scaling scale-dependent physical quantities inherent in the rupture

As noted elsewhere (Ohnaka and Shen, 1999[9]), there are in general two classes of physical quantities inherent in the shear rupture: scale-dependent quantities and scale-independent quantities. The scale-dependent quantities include the breakdown zone size and its duration (breakdown time), the nucleation zone size and its duration, the apparent shear rupture energy, the slip acceleration, and the cutoff frequency of the power spectral density of the slip acceleration versus time record observed at a position on the fault. These scale-dependent physical quantities can be treated unifyingly in quantitative terms by formulating the constitutive law for the shear rupture as a slip-dependent law so as to meet the physical principles and constraints to be imposed on the law. In fact, those scale-dependent quantities are expressed theoretically in terms of slip-dependent constitutive law parameters including D_c in the framework of fracture mechanics. For instance, the breakdown zone size X_c and the critical size $2L_c$ (L_c, half length) of the nucleation zone are expressed in terms of $\Delta\tau_b$ and D_c (Ohnaka and Yamashita, 1989[10]; Ohnaka and Shen, 1999[9]; Ohnaka, 2000[8])

$$X_c = L_c = (1/\mathrm{k})(\mu/\Delta\tau_b)D_c \quad (7)$$

where k is a dimensionless parameter depending on the rupture velocity, and μ is the rigidity. The apparent shear rupture energy G_c defined by (Palmer and Rice, 1973[12])

$$G_c = \int_0^{D_c} [\tau(D) - \tau_r] dD \quad (8)$$

is also expressed in terms of $\Delta\tau_b$ and D_c as (Ohnaka and Yamashita, 1989[10])

$$G_c = \tfrac{1}{2}\Gamma\Delta\tau_b D_c \quad (9)$$

where Γ is a dimensionless parameter. The peak slip acceleration $\ddot{D}_{max}$ is expressed as (Ida, 1973[5]; Ohnaka and Yamashita, 1989[10])

$$\ddot{D}_{max} = \frac{\Gamma^2 \phi''_{max}}{\pi^4}\left(\frac{V}{C(V)}\frac{\Delta\tau_b}{\mu}\right)^2 \frac{1}{D_c} \quad (10)$$

where ϕ''_{max} is the dimensionless peak acceleration, and $C(V)$ is a function of the rupture velocity V. We thus find that the scale dependence of scale-dependent physical quantities is commonly ascribed to the scale-dependence of the scale-dependent constitutive parameter D_c. We also find from equation (3) that the scale-dependence of D_c scales with the characteristic length λ_c representing geometric irregularity of the rupturing surfaces.

Characteristic length scales representing geometric irregularity of the fault surfaces

The above discussion indicates that geometric irregularity of the rupturing surfaces plays a fundamental role in scaling scale-dependent physical quantities inherent in the shear rupture. The reason for this may be understood from the following consideration. Real rupture surfaces of heterogeneous materials cannot be plane, but they exhibit geometric irregularity. During the shear rupture, slip displacement proceeds on the rupturing surfaces in the breakdown zone behind the propagating front of the rupture, and hence the rupturing surfaces are in mutual contact and are interacting during the breakdown process. This indicates that the shear rupture process is severely affected by geometric irregularity of the rupturing surfaces, and that the critical slip displacement D_c required for the cohesive zone behind the propagating rupture front to break down is greatly influenced by the geometric irregularity. Since D_c is one of the constitutive law parameters, the effect of the geometric irregularity needs to be incorporated in the constitutive law.

If the geometric property of the rupture surfaces is quantified properly in terms of a simple parameter, the effect of the geometric irregularity can easily be incorporated into the law. Irregular rupture surfaces of heterogeneous materials in general exhibit self-similarity; however, the self-similarity cannot be at all scales. The slipping process during the breakdown is the process that smoothes away geometric irregularity of the rupturing surfaces, and hence the self-similarity is necessarily at finite scales over limited bandwidths. This means that there is at least one characteristic length scale representing geometric irregularity of the rupture surfaces. The characteristic length λ_c is defined as the critical wavelength beyond which geometric irregularity of the rupture surfaces no longer exhibits the self-similarity. More generally, when a rupture surface has band-limited self-similarity (or fractal nature), a different fractal dimension can be calculated for each band, and a characteristic length λ_c can be defined as the corner wavelength that separates the neighboring two bands with different fractal dimensions. Note that the characteristic length defined as such represents a predominant wavelength component of geometric irregularity of the rupture surface.

Geometric irregularity of the rupture surface may thus in general be quantified and characterized in terms of the two parameters: the fractal dimension of each band, and the characteristic length defined as the corner wavelength that separates the neighboring two bands. Of these two parameters, it has been found that it is the characteristic length λ_c that plays a critical role in scaling the scale-dependent constitutive parameter D_c (see equation (3)). This leads to the important conclusion that the characteristic length departed from the self-similarity plays a key role in scaling scale-dependent physical quantities inherent in the shear rupture.

Characteristic length scales inferred for earthquakes

The above conclusion that the characteristic length plays a key role in scaling scale-dependent physical quantities inherent in the rupture, poses a question about how large the characteristic lengths for earthquakes are. Natural faults contain a wide range of characteristic length scales departed from the self-similarity. For instance, the self-similarity of natural faults is limited by the depth of seismogenic layer and fault segment size (e.g., Aki, 1992[2], 1996[3]; Knopoff, 1996[6]). The earthquake generation process and its eventual size are necessarily prescribed and characterized by these macroscopic length scales (Shimazaki, 1986[17]; Scholz, 1982[15], 1994[16]; Romanowicz, 1992[14]; Matsu'ura and Sato, 1997[7]). However, scale-dependent physical quantities such as X_c, L_c, G_c, and $\ddot{D}_{max}$ are controlled by a smaller scale of the characteristic lengths on the fault. High resistance to rupture growth will be attained at portions of fault bend or stepover, at interlocking asperities on the fault surfaces with topographic irregularity, and/or at portions of adhesion (or cohesion) healed between the mating fault surfaces during the inter-seismic period. A patch of such high resistance to rupture growth on a fault may act as a barrier against earthquake rupture. If such a patch is tough enough to sustain an adequate amount of the elastic strain energy stored in the surrounding medium, and if it is broken down as a consequence of tectonic loading, it will act as a source of energy supply for the spontaneous rupture. The size of such a patch, which is small compared with the depth of seismogenic layer and fault segment size, is also a candidate for the characteristic length scale departed from the self-similarity. The scale dependence of physical quantities such as X_c, L_c, G_c, and $\ddot{D}_{max}$ depends on how large the size of such a strong patch that is broken down on the fault is. For instance, G_c is written in terms of λ_c as follows

$$G_c = \frac{\Gamma}{2}\left(\frac{1}{\beta}\right)^{\frac{1}{M}}\left(\frac{\Delta\tau_b}{\tau_p}\right)^{\frac{M+1}{M}}\tau_p\lambda_c \tag{11}$$

or equivalently

$$G_c = \frac{\Gamma\beta}{2}\left(\frac{D_c}{\lambda_c}\right)^{M+1}\tau_p\lambda_c \tag{12}$$

That these relations are justified has been confirmed by laboratory data on both frictional slip failure and shear fracture of intact rock. It has also been checked that they are applicable to earthquake data.

The characteristic length λ_c can be inferred for earthquakes under the assumption that relations (3) and (4) can be extrapolated to the earthquake rupture. We have from (4) that $1/m$ ($=\lambda_c/D_c$) $\cong$ 1.5 when $\Delta\tau_b/\tau_p = 1$ (complete stress drop), $1/m = 10$ when $\Delta\tau_b/\tau_p = 0.1$, $1/m \cong 70$ when $\Delta\tau_b/\tau_p = 0.01$, and $1/m = 454$ when $\Delta\tau_b/\tau_p = 0.001$. It will be unrealistic to assume that $\tau_p \geq$ 1000 MPa at depths in the brittle seismogenic layer, because τ_p for intact granite tested at simulated crustal conditions in the laboratory does not exceed 1000 MPa. More specifically, if τ_p = 100 MPa is assumed under the constraint that $\Delta\tau_b \leq \tau_p$, $\Delta\tau_b/\tau_p$ takes a value ranging from 1 to 0.004 for Californian earthquakes analyzed by Papageorgiou and Aki (1983[13]), and Ellsworth and Beroza (1995[4]). In this case, λ_c/D_c has a value ranging from 1.5 to 144. If $k = 3$, μ = 30000 MPa, and $\Delta\tau_b$ = 10 MPa are further assumed, we have from (7) that $X_c \cong 10^3 D_c$, and from (3) and (4) that $\lambda_c = 10D_c$. Hence, we have $Xc \cong 10^2\lambda_c$. The relation $X_c \cong 10^3 D_c$ means that for the cohesive zone size of, for instance, 1 km to break down, the critical amount of slip of 1 m is required. The relation $X_c \cong 10^2\lambda_c$ indicates that the predominant wavelength component of 10 m is contained in geometric irregularity of the rupture surfaces for the cohesive zone size of 1 km. This may be paraphrased as follows. On a fault having the characteristic length (predominant wavelength) of 10 m, the amount of D_c becomes 1 m under the stress condition that $\Delta\tau_b/\tau_p = 0.1$.

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