On the slip weakening behavior in rate- and state-dependent constitutive laws

Massimo Cocco[1] and Andrea Bizzarri[1,2]

(1) Istituto Nazionale di Geofisica e Vulcanologia, Rome, Italia(e-mail: cocco@ingrm.it cocco@ingv.it, phone +39-06 51860401; fax: +39-06 51860507). (2) Università di Bologna, Dipartimento di Fisica, Bologna, Italy (phone: +39-051 2095026).

Abstract

We study the dynamic propagation of a 2D in-plane crack using a finite difference approach to compare slip weakening and rate- and state-dependent constitutive laws for equivalent sets of initial parameters. Our modeling results confirm that the dynamic solution obtained by using a rate- and state-dependent law implies a slip dependence of the dynamic traction. The shape of the resulting slip weakening curve displays a slip hardening phase preceding the traction drop (weakening phase) and an equivalent critical slip weakening distance (D_o^{eq}). In these simulations the characteristic length for scaling the dynamic parameters is the distance L, which controls the state variable evolution. This implies that the nucleation patch depends on L and not on D_o^{eq}. We show that the adopted constitutive parameters A, B and L control the traction dependence of slip, since D_o^{eq}, the weakening rate as well as the slip-hardening phase depend on these constitutive parameters.

Introduction

The dynamic problem of the crack propagation and arrest has been studied in the literature by using 2–D solutions (Andrews, 1976a [1]; Cochard & Madariaga, 1996 [2], and references therein), as well as by 3–D numerical approaches (see Fukuyama & Madariaga, 1998 [3]; Madariaga et al., 1998 [4], among different others). Bizzarri et al. (2000) [5] have compared the 2-D in-plane solutions resulting from a boundary integral equation and a finite difference algorithms and concluded that, if a careful control of resolution and stability is performed, the two methods yields identical solutions and similar time histories of dynamic source parameters. Although less applicable to real faults, a 2-D solution offers the advantage of allowing the use of sophisticated constitutive laws. In the present study we investigate the dynamic propagation of a 2–D in–plane crack (Andrews, 1976b [6]; Andrews & Ben–Zion, 1997 [7]) by using the same finite difference numerical approach, which allows the use of different frictional laws and heterogeneous distribution of constitutive parameters.

The solution of the dynamic rupture problem requires the use of a constitutive law that relates the total dynamic traction to fault friction. In the literature different constitutive equations have been proposed, which can be grouped in two main classes: the slip dependent (see Andrews, 1976a, b [1,6]; Matsu'ura et al., 1992 [8]; Ohnaka and Shen, 1999 [9], and references therein) and the rate- and state-dependent laws (Dieterich, 1986 [10]; Ruina, 1983 [11]; Marone, 1998 [12] and references therein). The former assumes that friction (or total traction) is a function of the fault slip only; while the latter implies that the friction is a function of slip velocity and state variables. Okubo (1989) [13] pointed out that the use of a rate- and state-dependent friction law implies a slip dependence of the dynamic traction. This result has been also previously observed by different authors either in laboratory experiments or in theoretical models. It is important to emphasize that, if the aim is to reproduce the rupture history during a single earthquake, a slip–weakening law is sufficient. However, if we analyze the mechanical conditions of a fault during its seismic cycle, thus a rate- and state- formalism provide a better description of the state of stress on the rupture plane.

This consideration has important implications on the nucleation process. In recent years, the problem of rupture initiation has been studied either theoretically or experimentally, and there were few attempts in constraining such dynamic features by using recorded seismograms (e. g. Ellsworth & Beroza, 1995 [14]). Recently, Campillo & Ionescu (1997) [15] have derived an analytical expression for the displacement arising from an anti–plane shear crack under a slip–weakening governing equation. According to Ionescu & Campillo (1999) [16] the initial value of friction is a key parameter in controlling the nucleation process. In the present study we aim to point out that the solution resulting from the rate– and state–dependent law involves more general behaviors during the nucleation stage.

The numerical procedure

In this work we solve the elastodynamic equation

$$\rho\, \ddot{u}_i = \sigma_{ij,j} + f_i \qquad (1)$$

for a 2–D in–plane shear crack for which the displacement and the shear traction depend on time and one spatial coordinate and neglecting the body forces ($f_i = 0$). In (1) ρ is the mass density, u the displacement and $\sigma_{ij,j}$ is the spatial derivative of the stress tensor. In particular, we assume that the crack propagates along x_1 in the $x_3 = 0$ plane. The medium is supposed to be infinite, homogeneous and elastic everywhere except along the fracture line. We solve equation (1) by using a finite difference (FD) approach originally proposed by Andrews (1973) [17] and recently used by Andrews & Ben–Zion (1997) [7] and implemented by Bizzarri et al. (2000) [5] to include spatially variable constitutive parameters and to consider different constitutive laws. We solve the discretized equations deriving from (1):

$$\begin{aligned} \rho \frac{\partial}{\partial t} \dot{u}_1 &= \frac{\partial}{\partial x_1} \Sigma_{11} + \frac{\partial}{\partial x_2} \Sigma_{12} \\ \rho \frac{\partial}{\partial t} \dot{u}_2 &= \frac{\partial}{\partial x_1} \Sigma_{12} + \frac{\partial}{\partial x_2} \Sigma_{22} \end{aligned} \qquad (2)$$

In (2) $\dot{u}_i$ indicate the time derivative of the slip and Σ_{ij} are the stress tensor components (which is the elastic stress tensor since we do not consider any viscous term, see Andrews, 1973 [17] for details). The x_1–x_2 plane is linear elastic everywhere except along the fault line x_1, where the constitutive equation is introduced. The fault is described by a number of split nodes coupled to each other by the constitutive relations. The solutions of (2) are stepped through time by calculating the net force acting on every node, by adjourning the velocities and the displacements and by recalculating the internal force that every element exercises on its nodes.

In this approach a grid of nodes is introduced and each node is a vertex of an equilateral triangle; the choice of triangles, instead of other geometrical figures as rectangles for instance, has been made in order to increase the numerical efficiency. All variables are defined in each node and therefore it is possible to consider the entire medium surrounding the fault line, eventually accounting for material heterogeneity.

The constitutive equations

The more general formulation for a constitutive law involves the definition of the maximum frictional strength on the fault plane, which is a function of several constitutive parameters

$$\tau = f(u, \dot{u}, \sigma_n^{eff}, c_e, \lambda_c, T, \Psi) \tag{3}$$

where u is the slip, $\dot{u}$ the slip velocity, σ_n^{eff} the effective normal stress (which includes the effects of pore fluids), c_e the chemical effect of the fluid pressure, λ_c a parameter describing the geometric characteristics of the fault surface (roughness, fault gouge, etc.), T the temperature and finally $\Psi \equiv (\Psi_1, \ldots, \Psi_N)$ is the state variable (Dieterich, 1986 [10]; Ruina, 1983 [11]). Equation (3) is usually named the governing equation. The most important effect of the state variables Ψ• is that they obey a specified temporal evolution law, which is established by a set of differential equations associated with the governing relation (3):

$$\frac{\mathrm{d}}{\mathrm{d}t}\Psi_i = g_i(u, \dot{u}, \sigma_n^{eff}, c_e, \lambda_c, T, \mathbf{\Psi}) \qquad i = 1, \ldots, N \tag{4}$$

where the functions g_i also depend on the constitutive parameters previously defined. Equations (4) are usually called evolution equations. In this study we have used two different and widely applied constitutive relations to study the dynamics of a crack propagation: a slip–weakening law and a rate– and state–dependent friction law.

The slip–weakening law

The SW law assumes that the frictional strength is a function only of the slip u and that in particular it results:

$$\tau = \begin{cases} \tau_u - (\tau_u - \tau_f)\dfrac{u}{d_0} & , u < d_0 \\ \tau_f & , u \geq d_0 \end{cases} \tag{5}$$

The constitutive parameters used in this law are (Andrews 1976a [1], b [6]): the initial shear stress τ_0, the yield stress τ_u, the frictional kinetic level τ_f and the characteristic slip d_0. The length d_0 allows to identify the cohesive zone, defined as the region where the fracture energy is released (see Fig. 1a). The fault strength reduction for increasing slip is physically reasonable, because it accounts for the diminution of friction due to the progressive abrasion of the microscopic asperities existing on the surfaces in mutual contact. Among the different formulations of the slip–dependent law proposed in the literature, their main difference concerns the weakening rate, that is the rate of friction decrease as a function of slip (i. e. the slope of the curves in Fig. 1). Matsu'ura et al. (1992) [8] and Shibazaki & Matsu'ura (1998) [18] use an exponential decrease of friction τ for increasing slip, instead of linear decrease (constant negative slope) as used by Andrews and shown in Fig. 1(a). According to these results we emphasize that, if the slip–weakening curve has a variable slope, different frictional behaviors are expected. In particular, if the initial slope is positive, a slip–hardening phase precedes the slip–weakening behavior (see Fig. 1b). The simplest SW model is shown in Fig.1a, which implies a constant weakening rate during the dynamic process. A slip–dependent constitutive law with varying weakening rate can be considered as a more heterogeneous version of the classical SW law proposed by Andrews (1976a, b).

The rate– and state–dependent friction laws

Rate– and state–dependent friction laws represent a more general form of constitutive relations than the slip–weakening law. They were proposed to explain stick–slip behavior during laboratory experiments. In these laws the friction depends on the slip rate $\dot{u}$, the state variable Ψ,• and the effective normal stress σ_n^{eff}. We use these constitutive relations assuming that the normal stress σ_n^{eff} is constant and adopting the Dieterich–Ruina (DR) law (Dieterich, 1986 [10] and references therein) in its "original" formulation, that is described by the following equations:

$$\begin{cases} \tau = \left[\mu_* - a \ln\left(\frac{v_*}{v} + 1 \right) + b \ln\left(\frac{\Phi v_*}{L} + 1 \right) \right] \sigma_n^{eff} \\ \frac{\mathrm{d}}{\mathrm{d}t} \Phi = 1 - \frac{\Phi v}{L} \end{cases} \qquad (6)$$

where μ*• and v* are arbitrary reference values of the friction coefficient and of the slip velocity, respectively; a, b and L are the constitutive parameters and Φ is the state variable. In the DR model the state variable has the physical meaning of an average contact time between the sliding surfaces (Ruina, 1983 [11]). The evolution equation (6) is usually called the slowness (or ageing) law, and it includes true ageing. In the low velocity limit the state variable Φ in the DR formulation (6) evolves as linear increase in time, yielding to a re–strengthening process.

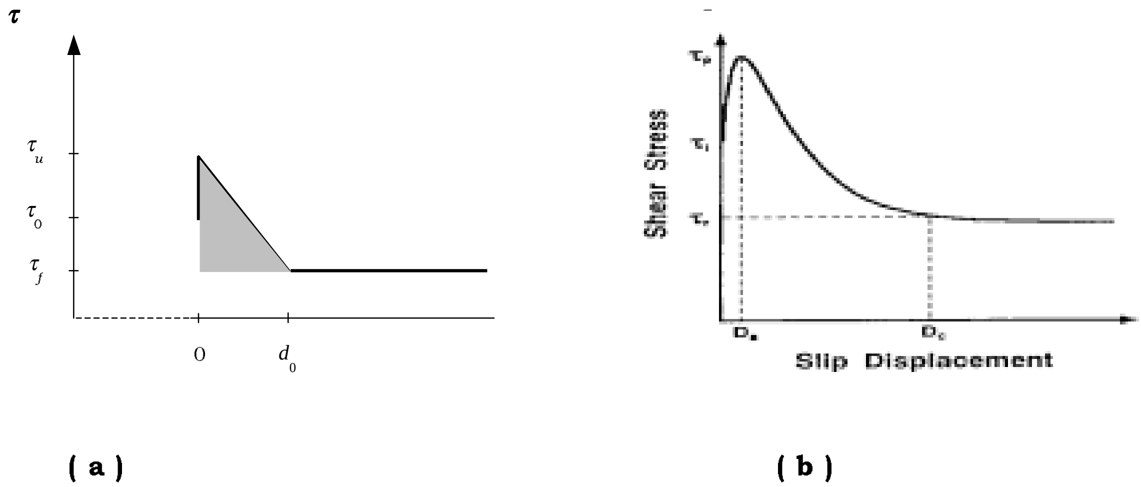

Figure 1: The slip–weakening model. (a) Constitutive law as introduced by Andrews (1976a, b). The shaded area represents the released fracture energy. do is the critical slip weakening distance. (b) Slip–weakening law proposed by Ohnaka & Yamashita (1989). Dc is the critical slip weakening distance.

The constitutive parameter $A = a\sigma_n^{eff}$ represents an instantaneous rate sensitivity: that is, the direct effect of the friction after a sudden change in slip velocity. $B = b\sigma_n^{eff}$ controls instead the evolution of the state variable. The characteristic distance L is the length over which the surface slips before the motion approximates the steady state sliding.
The quantity B – A plays a very important role in the sliding process, because it determines the sign of (d/dt)τ• $_{ss}$(v), that is, of the frictional rate in the stationary conditions. Stability analyses have pointed out that for (d/dt)τ. $_{ss} > 0$ (i. e. for $B - A < 0$) the sliding is stable, while for (d/dt)τ. $_{ss} < 0$ (i. e. for $B - A > 0$) the sliding is unstable. The first behavior (aseismic) is defined in the literature as velocity strengthening, while the second (potentially seismic) as velocity weakening.

Modeling Results

In this study we have compared the rupture evolution resulting from a slip–weakening and a rate– and state–dependent friction law. This comparison shows that, despite the different constitutive formulations, similar behaviors are simulated during the rupture propagation and arrest. We observe a crack tip bifurcation and a jump in rupture velocity (approaching the P–wave speed) also with the Dieterich–Ruina (DR) law. The rupture arrest at a barrier (high strength zone) and the barrier–healing mechanism are also reproduced by this law. However, this constitutive formulation allows the simulation of a more general and complex variety of rupture behaviors. By assuming different heterogeneous distribution of the initial constitutive parameters, we were able to model a barrier–healing as well as a self–healing process. This result suggests that, if the heterogeneity of the constitutive parameters is taken into account, the different healing mechanisms can be simulated.

As previously mentioned, our modeling results clearly show that the dynamic solution obtained by using a rate- and state-dependent friction law shows a slip dependence of the dynamic traction, which may contain a slip weakening behavior preceded by a slip hardening phase (see Fig.2). Because our characteristic length intrinsic of the dynamic problem is the parameter L (see eq. 6), we define an equivalent critical slip weakening distance D_o^{eq} as shown in Fig.2. The nucleation patch in our constitutive model depends on L, and not on D_o^{eq}.

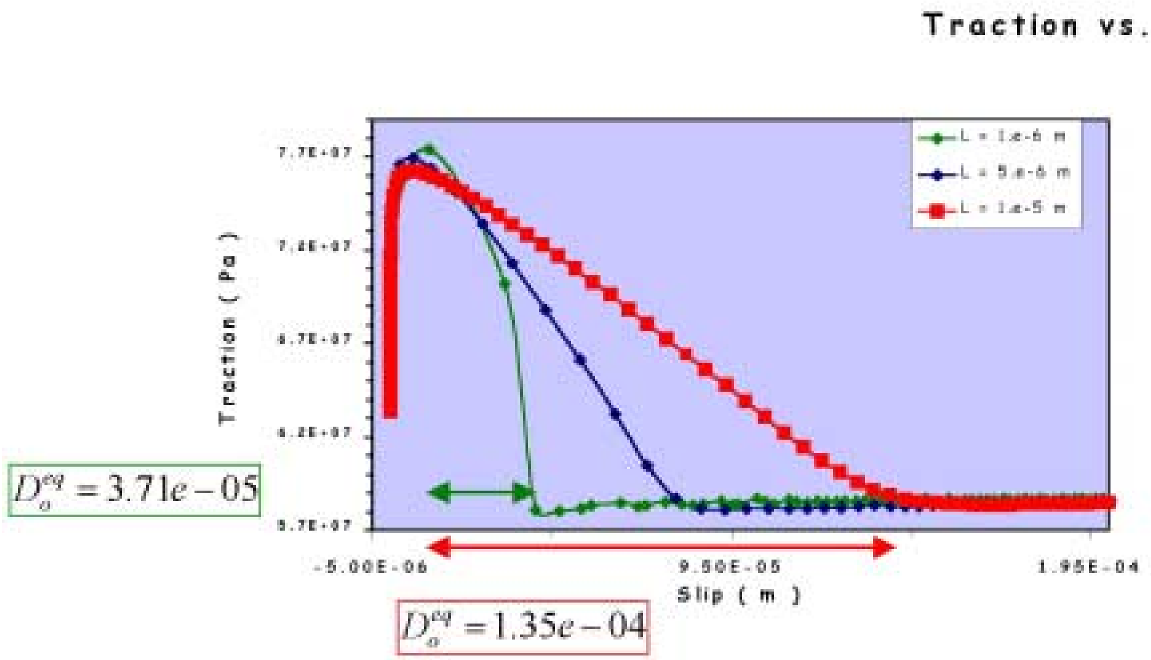

Figure 2. Slip-weakening behavior resulting from rate- and state-dependent frictional law for different values of the parameter L. Although the initial slip-hardening behavior is not evident in these simulations, it is present in many others. The simulations here presented clearly show that using different values of the L parameter implies different slip weakening distances D_o^{eq} and different weakening rates.

Our simulations show that the adopted constitutive parameters A, B and L control the traction dependence on slip. In particular, we emphasize that for the same L value but using different B and A, the equivalent critical slip weakening distance as well as the slip-hardening phase changes considerably. For particular set of constitutive parameters the slip-hardening phase can be absent (see Fig.2). In general, it is possible to have different equivalent (or apparent) critical slip weakening distances for the same characteristic length of the dynamic problem. Moreover, the weakening rate depends on L. On the contrary, D_o^{eq} does not depend on the initial value of the state variable.

These results have important implications since they allow to explain large critical slip weakening distances as resulting from dynamic consistent waveform inversions, but small nucleation patches. Moreover, they provide a physical interpretation of the variation of the weakening rate and of the initial slope of the weakening curve, which have been recently proposed to control the nucleation duration.

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References

[1] Andrews D. J., 1976a, *Rupture propagation with finite stress in antiplane strain*, J. Geophys. Res., **81**, No. 20, pp. 3575 – 3582

[2] Cochard A., Madariaga R.,1996, *Complexity of seismicity due to highly rate dependent* friction, J. Geophys. Res., 101, No. B11, pp. 25321 – 25336

[3] Fukuyama E., Madariaga R., 1998, *Rupture dynamics of a planar fault in a 3D elastic medium: rate – and slip – weakening friction*, Bull. Seism. Soc. Am., **88**, No. 1, pp. 1 – 17

[4] Madariaga R., Olsen K., Archuleta R, 1998, *Modeling dynamic rupture in a 3D earthquake fault model*, Bull. Seism. Soc. Am., **88**, No. 5, pp. 1182 – 1197ù

[5] Bizzarri A., Cocco, M., Andrews D.J. and E. Boschi, 2000, Solving dynamic rupture problem with different numerical approaches and constitutive laws, accepted for publication on G.J.Int.

[6] Andrews D. J., 1976b, *Rupture velocity of plane strain shear cracks*, J. Geophys. Res., **81**, No. 32, pp. 5679 – 568

[7] Andrews D. J., Ben – Zion Y., 1997, *Wrinkle – like slip pulse on a fault between different materials*, J. Geophys. Res., **102**, No. B1, pp. 553 – 571

[8] Matsu'ura M., Kataoka H., Shibazaki B,.1992, *Slip – dependent friction law and nucleation processes in earthquake rupture*, Tectonophysics, **211**, pp. 135 – 148

[9] Ohnaka M., Shen L., 1999, *Scaling of the shear rupture process from nucleation to dynamic propagation: implications of geometric irregularity of the rupturing surfaces*, J. Geophys. Res., **104**, No. B1, pp. 817 – 844

[10] Dieterich J. H, 1986, *A model for the nucleation of earthquake slip*, Earthquake Source Mechanics, Geophysical Monograph, 37, Maurice Ewing Series, 6, edited by S. Das, J. Boatwright and C. H. Scholz, Am. Geophys. Union, Washington D. C., pp. 37 – 47

[11] Ruina A. L., 1983, *Slip instability and state variable friction laws*, J. Geophys. Res., **88**, No. B12, pp. 10359 – 10370

[12] Marone C., 1998, *Laboratory – derived friction laws and their application to seismic faulting*, Annu. Rev. Earth Planet Sci., 26, pp. 643 – 696

[13] Okubo P. G., 1989, *Dynamic rupture modeling with laboratory – derived constitutive relations*, J. Geophys. Res., **94**, No. B9, pp. 12321 – 12335

[14] Ellsworth W. L., Beroza G. C., 1995, *Seismic evidence for an earthquake nucleation phase*, Science, **268**, pp. 851 – 855

[15] Campillo M., Ionescu I. R., 1997, *Initiation of antiplane shear instability under slip dependent friction*, J. Geophys. Res., **102**, No. B9, pp. 20363 – 20371

[16] Ionescu I. R., Campillo M., 1999, *Influence of the shape of the friction law and fault finiteness on the duration of initiation*, J. Geophys. Res., **104**, No. B2, pp. 3013 – 3024

[17] Andrews D. J., 1973, *A numerical study of tectonic stress release by underground explosions*, Bull. Seism. Soc. Am., **63**, No. 4, pp. 1375 – 1391

[18] Shibazaki B., Matsu'ura M., 1998, *Transition process from nucleation to high – speed rupture propagation: scaling from stick – slip experiments to natural earthquakes*, Geophys. J. Intl., **132**, pp. 14 – 30