Thermal Convection Analysis in a Rotating Shell by a Parallel FEM - Development of a Thermal-Hydraulic Subsystem of GeoFEM -

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Abstract

For investigation of fluid dynamics in the Earth's outer core and mantle, we have been developing thermal-hydraulic subsystem of GeoFEM, which gives a parallel finite element method (FEM) platform. This subsystem is a three dimensional and time-dependent simulation code for an incompressible fluid with the Boussinesq approximation. This subsystem is tested on an analysis of lid-driven convection in a cubic cavity. To investigate focus on the Earth's core dynamics in this study, we carry out a thermal convection analysis in a rotating spherical shell which is modeled on the Earth's outer core. The results of the simulation are compared to a simulation using the spectral method. In both cases, the results of the convection pattern show that three pairs of convection columns are formed and that these columns propagate to westward.

Introduction

It is widely accepted that Earth's outer core consists of an electrically conducting fluid and that the geomagnetic field is generated by a motion of the fluid core; this process is known as a dynamo process. For understanding the dynamics of the outer core and the dynamo process, numerical simulation has a very important role. In the last few years, several MHD dynamo simulations in a rotating spherical shell have reproduced successfully some of the basic properties of the dynamo processes (Glatzmaier 1995a[2], 1995b[3]; Kuang and Bloxham, 1997[4], 1999[5]; Christensen, 1999[1]). Most of the studies have been obtained by the spectral harmonics expansion in the azimuthal and elevation direction. In this study, we have developed a three dimensional and time-dependent thermal-hydraulic subsystem of GeoFEM, which gives a platform for parallel FEM, and have verified this code by an analysis of the lid-driven convection in a cubic cavity. Then, We have carried out a thermal convection analysis in a rotating spherical shell and have compared the results to the same analysis by the spectral method. The results shows that similar patterns are formed in both cases.

Physical model and numerical method

This subsystem is a parallel FEM scheme for incompressible fluid with the Boussinesq approximation. In this subsystem, The momentum equation, the heat conduction equation, and the continuous equation for the fluid are solved simultaneously. In the simulation for the rotating spherical shell, The Coriolis force and self gravity for source of the buoyancy force are applied. To obtain non-dimensional forms of the basic equations, we choose the width of the fluid shell L and thermal diffusion time L^2/κ, where κ is the thermal diffusivity, as the length scale and the time scale, respectively. The temperature difference ΔT between the inner boundary and the outer boundary of the shell is chosen as the temperature scale. The normalized basic equations in a rotating flame are given as follows,

$$\begin{aligned} v_{i,i} &= 0, \\ \partial_t v_i + v_j v_{i,j} &= -P_{,i} + P_r v_{i,jj} - P_r \sqrt{T_a} e_{ijk} \Omega_j v_k + P_r R_a (T - T_0) r_i, \text{ and} \\ \partial_t T + v_j T_{,j} &= T_{,jj}, \end{aligned} \tag{1}$$

where v_i, P, T, T_0, $\Omega_i = \hat{z}$, r_i, and e_{ijk} are the velocity, the pressure, the temperature, reference of the temperature, the unit vector of the Earth's rotation, the position vector, and the permutation symbol, respectively. Three dimensionless numbers appear in the basic equations; that is, the Prandtl number P_r, the Taylor number T_a, and the Rayleigh number R_a are defined by,

$$\begin{aligned} P_r &= \frac{\nu}{\kappa}, \\ T_a &= \left(\frac{2\Omega L^2}{\nu} \right)^2, \text{ and} \\ R_a &= \frac{\alpha g \Delta T L^3}{\kappa \nu}, \end{aligned} \tag{2}$$

where ν, Ω, α, g are the kinetic viscosity, the angular velocity of the Earth's rotation, the thermal expansion, and gravity. The velocity, the temperature, and the pressure are interpolated by tri-linear function in each hexahedron element. For the time integration, SMAC scheme is applied; that is, time integration of the temperature and the velocity is calculated by the Adams-Bashforth scheme, and the pressure is solved by the parallel iterative solver.

This subsystem is verified by the analysis of a convection in a lid-driven cavity at the Reynolds number $R_e = 1000$. In this analysis, the Coriolis force and the buoyancy force are not considered.

Geometry of the shell and dimensionless numbers

We obtain a simulation of the thermal convection in a rotating spherical shell modeled on the Earth's outer core. The ratio of the inner radius to the outer radius of the shell is set to be 0.4; the shell is assumed to be filled with incompressible fluid that is subjected to thermally driven convection with the Boussinesq approximation. In this problem, self gravity is applied as source of the buoyancy force. We apply a quite simple model; that is, the constant diffusivities are applied and internal heat source is not considered in this study. Because of the limitation of the computational power, we apply the dimensionless numbers which are far from the estimated properties of the Earth's core in the present study; that is, the P_r is set to be 1, with the $T_a = 2.5 \times 10^5$, and R_a set to be 1.5×10^4. We apply the non-slip boundary condition for the velocity field on the inner and outer boundaries, and set the temperature to be 1 on the inner boundary and to be 0 on the outer boundary.

Results of the simulation

The results of the simulation of the convection pattern show that three pairs of the Taylor columns are dominantly formed (see Figure 1) and that these columns propagate to the westward in the quasi-steady state of the simulation. We compare these results to that of the simulation with same parameters by the spectral method (Matsui, 1999[6]). Although different initial values of the temperature are set in these two cases, the results of the convection patterns are similar between two cases. On the other hand, the kinetic energy in the shell is about 90% of that in the case using the spectral method. We consider that this discrepancy is caused by the difference of the spatial resolution and of estimation method of the kinetic energy between these two simulations.

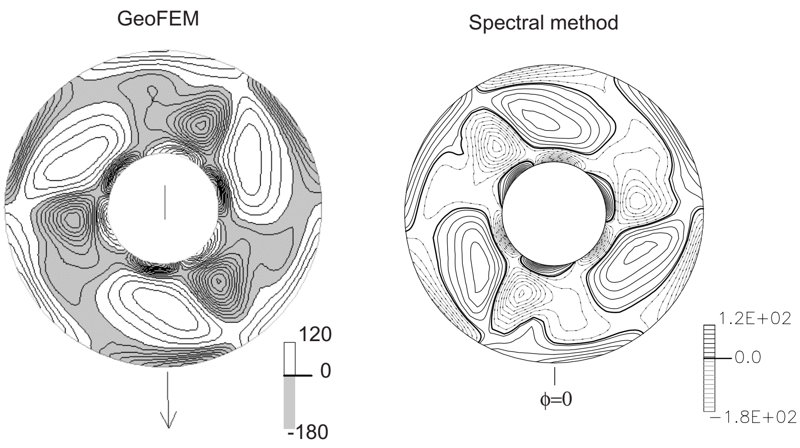

Figure 1: Intensity of the z-component of the vorticity at a cross section of z = 0.34 at a quasi-steady state. Range of the contour map is from -180.0 to 120.0. The result by GeoFEM is given in left panel, and that by the spectral method is given in the right panel.

Conclusion

We have carried out a thermally driven convection analysis in a rotating spherical shell modeled on the Earth's outer core by the Thermal-Hydraulic Subsystem of GeoFEM, which gives a parallel FEM platform. We obtain the following results by comparing with the simulation results by the spectral method.

- The convection patterns at the quasi-steady state are similar to each other.
- The convection patterns are propagate to westward in both cases.
- Kinetic energy of the fluid by FEM is 90% of that by the spectral method.

To investigate the Earth's core dynamics and the geodynamo process, We have to simulate the motion of the fluid magnetohydrodynamically. Thus, development of a MHD code based on this subsystem is deferred for the further studies.

References

[1] Christensen, U., P. Olson and G.A. Glatzmaier, 1999, *Numerical modelling of the geodynamo: a systematic parameter study,* Geophys. J. Int. , **138**, 393-409.

[2] Glatzmaier, G.A. and P.H. Roberts, 1995a, *A three-dimensional self-consistent computer simulation of a geomagnetic field reversal,* Nature , **377**, 203-209.

[3] Glatzmaier, G.A. and P.H. Roberts, 1995b, *A three-dimensional convective dynamo solution with rotating and finitely conducting inner core and mantle,* Phys. Earth Planet. Inter., **91**, 63-75.

[4] Kuang, W. and J. Bloxham, 1997, *An Earth-like numerical dynamo model,* Nature, **389**, 371-374.

[5] Kuang, W. and J. Bloxham, 1999, *Numerical modeling of magnetohydrodynamic convection in a rapidly rotating spherical shell: weak and strong field dynamo action,* J. Comput. Phys., **153**, 51-81.

[6] Hiroaki Matsui, 1999, *Studies on the Basic Processes of Magnetic Field Generation Based on MHD Simulation in the Rotating Spherical Shell,* Ph.D. Thesis, Tohoku Univ.