Recurrent method of solving direct and inverse seismic problems

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Abstract

A method of solving inverse problems is proposed, based on the results of a direct dynamic problem of seismicity, presented in the form of recurrent relationships. The equations obtained are suitable for numerical calculations. The methods for determining for earthquake source parameters have been developed. A source of seismic waves modeled by a circular dislocations and source parameters may be found using the recurrent method. It should be noted that wave field values on a free surface are obtained as strict recurrent analytical relations.

Introduction

Studying spectral and dynamic parameters of earthquake sources is an essential condition to solve many problems of seismology. Physical properties of a medium as a function of stress must be studied in order to understand physical processes run near the source. In other words it is necessary to study the processes both of forming a source and propagation a fracture when stress decreases. We'll proceed from the theory that forming a fracture is a destruction process under the action of sliding stresses. It should be noted that many papers are devoted to the problem of earthquake source mechanisms [1,2]. Nevertheless, there is no any certain conception for interpretation of physical characteristics of a source. Besides, it is known that formation and extension of an earthquake source depend on its medium parameters. Thus the problem of simultaneous determining characteristics of a source and a medium is considered to be very important and urgent. Two problems to be solved are stated. First, advantages and disadvantages of some methods for determining parameters of an earthquake source are to be studied. Second, the recurrent method developed to solve dynamic problems of seismology is suggested for simultaneous determining characteristics of a source and a medium investigated.

Methods of azimuthal hodograph

Consider the method for determining characteristics of an earthquake source using azimuthal hodograph. The records of earthquakes with different source depth made at near-by and distant from an epicentre stations of Transcarpathian seismogenic zone are considered. Consider wave emission occurring during a certain moment of time T, i.e. maximal destruction takes place not in the very beginning but in a certain time interval. Azimuthal hodograph $t_{p_i} = f(\Delta = const, \varphi)$ which is of cosine form for source waves is considered to interpreted wave field induced by a movable source. For reflected, defracted and other waves the hodograph is a direct line parallel to the azimuth axis. This fact may be used as a criterion for wave separation. Using standard hodograph transform the run time of P-waves at all seismic stations into one epicentral distance $\Delta = const$.

$$t_{p_i}(\Delta = const) = t_p(\Delta st) - (t'_{p_i} - t''_{p_i}) - t_0 \pm f_k, \tag{1}$$

where $t_p(\Delta st)$ - arrival time of P-waves at a station with an epicentral distance Δst,

t_0 - time at first *P*-wave events in the source,

t'_{p_i}, t''_{p_i} - run time of P-waves run along standard hodograph at distances Δ and Δst,

f_k - "discrepancy".

The choice of an epicentral distance doesn't influence essentially the final results, as when source characteristics are calculated relative delay values of *P*-wave time delay for different azimuths are used; the length of source *L*, propagation velocities C and fracture direction φ are determined by maximal τ_{max} and minimal τ_{min} time delay of P_i wave:

$$L = V_p (\tau_{max} - \tau_{min}) / 2$$
$$C = V_p (\tau_{max} - \tau_{min}) / (\tau_{max} + \tau_{min}) \qquad (2)$$
$$A_z = 180^0 + A_{z\,max}$$

Fig.1 presents directions of fracture propagation for certain earthquakes occurred in Transcarpathian seismogenic zone in 1994-1996 (as shown in Table 1). As an example, azimuthal hodograph is constructed for a Hust earthquake of December 22, 1996 (Fig.2). According to Eq (2) its source characteristics are:

$$L = 4,5 km, C = 1,1 \frac{km}{c}, \varphi_1 = 3^0, \varphi_2 = 173^0$$

Fracture directions for earthquakes 1, 2, 3, 8, 9 (Fig.1) are not perpendicular to Oashski Fault; this characterises physical nature of forming this Fault. Fracture directions of a series of earthquakes are parallel to the above-mentioned Fault. Conclusions on local earthquakes in Transcarpathian seismogenic zone may be drawn mainly due to further investigations of fracture directions.

Table1. Catalogue of seismic events

| N	Data	Time	h, km	φ	λ	K	MSH	Region
1.	28/1/1994	22:47:44.50	6.0	48.1700N	23.1400E	6.6	1.1	Transcarpathians, Vynogradiv
2.	9/2/1994	22:25: 7.12	6.0	48.0386N	23.1448E	8.0	1.6	Transcarpathians, Vynogradiv
3.	9/2/1994	22:38:57.57	6.0	47.9844N	23.1251E	6.9	1.0	Transcarpathians, Vynogradiv
4.	9/9/1994	5:10:17.70	6.0	48.1051N	23.7279E	8.5	1.8	Transcarpathians, Ughil
5.	25/1/1995	10:48:43.86	1.4	48.3646N	22.8871E	7.5	1.5	Transcarpathians, Irshava
6.	24/4/1995	15: 3:41.44	6.0	48.1643N	23.6215E	7.6	1.4	Transcarpathians, Ughil
7.	24/4/1995	15:11:13.61	6.0	48.2317N	23.6611E	6.7	1.1	Transcarpathians, Ughil
8.	27/4/1995	8: 4:56:49	4.1	48.1816N	23.3256E	6.9	1.2	Transcarpathians, Hust
9.	26/5/1995	14:49:47.02	3.2	48.18.27N	23.3513E	7.3	1.4	Transcarpathians, Hust
10.	22/12/1995	14:58:38.54	5.6	48.1191N	23.4721E	7.7	1.4	Transcarpathians, on the east of Hust
11.	29/11/1996	21:38:15.39	3.6	48.3214N	23.3965E	5.9	0.7	Transcarpathians, on the north of N.Selyshche
12.	30/11/1996	13:11:32.04	6.0	48.3146N	23.6336E	7.6	1.4	Transcarpathians, on the west of Ust-Chorne
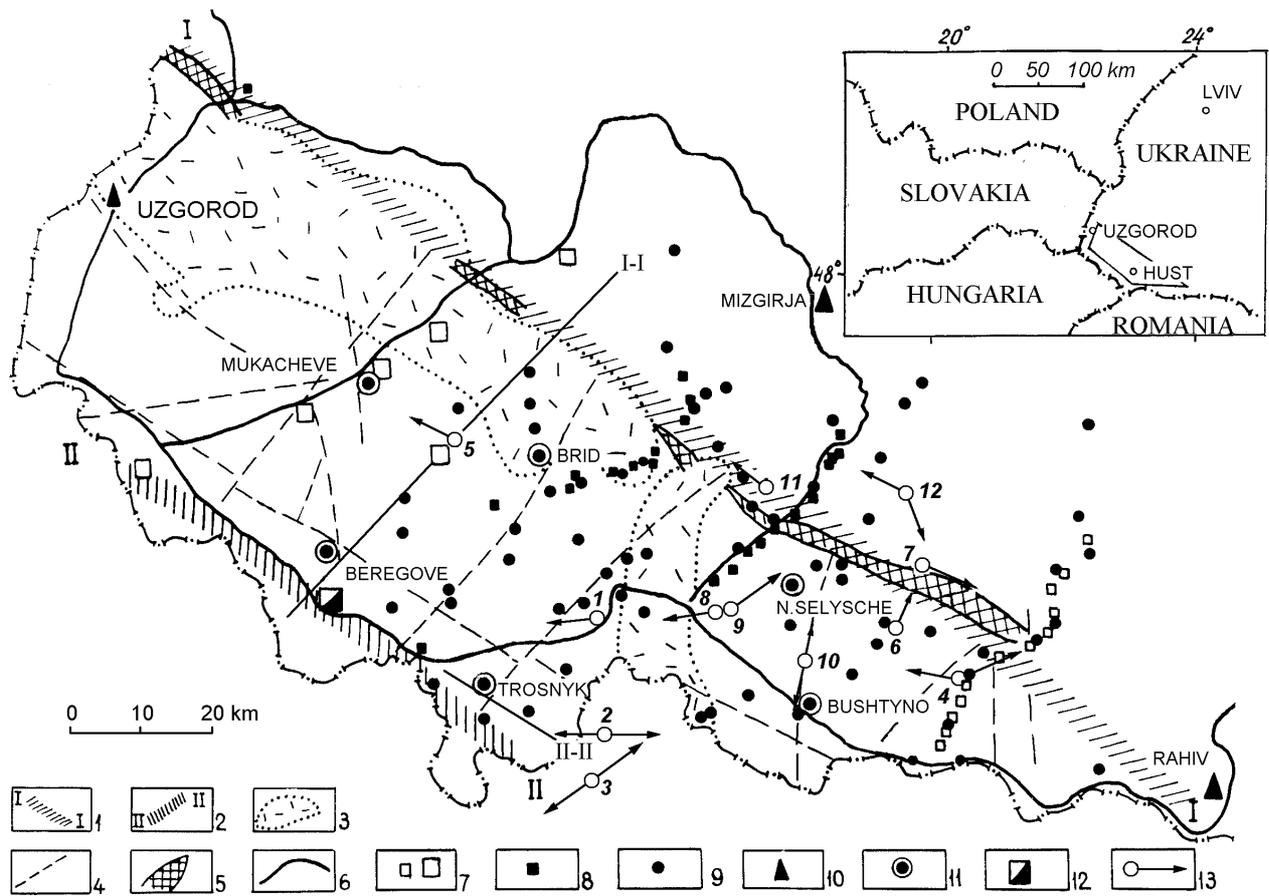
Figure1: Caprathian geodynamic polygon of the Carpathian Branch of the Institute of Geophysics of the National Academy of Sciences of Ukraine:
1) Transcarpathian deep fault; 2) Cis Pannonian deep fault; 3) Vyrgolat-Gutinska volcanic range; 4) pre-Neogene fundation of the Transcarpathian trough; 5) cliffs zone; 6) repeated levelling polygon; 7) fundamental levelling benchmark; 8) secular universal benchmark; 9) geomagnetic observation point; 10) regional seismic station; 11) regime geophysical stations; 12) mine; 13) earthquakes.
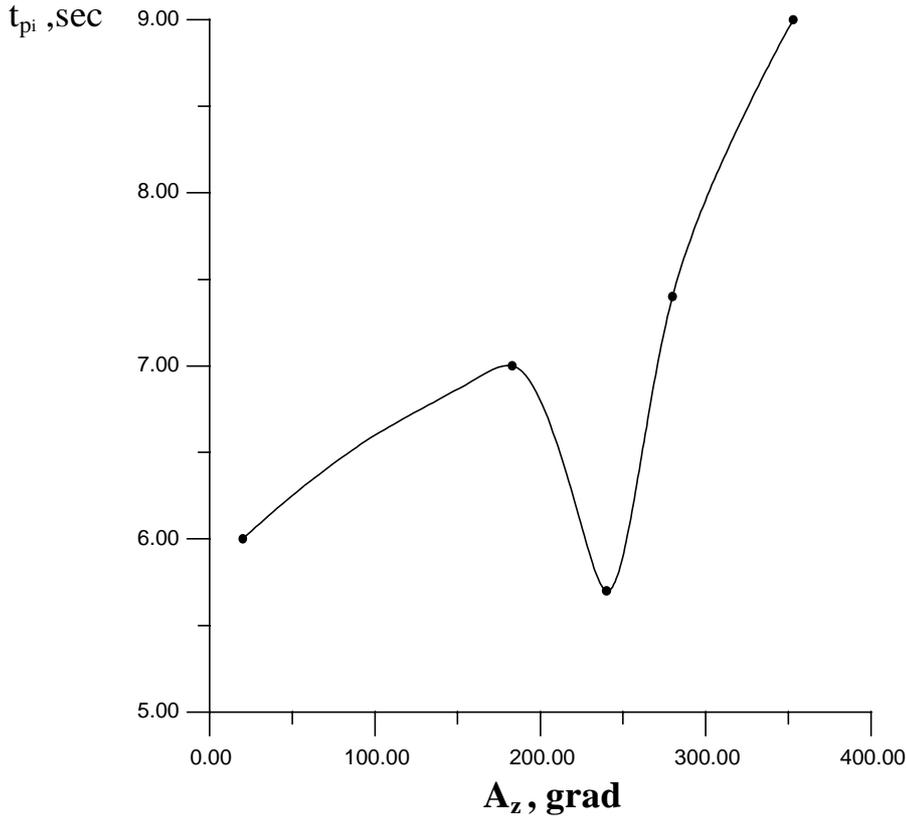
Figure 2: Azimuthal hodograph of wave P_1 for a Hust earthquake of December 22.1995.

Recurrent method

The recurrent method is suggested to determine velocities V_p, V_s of seismic waves, slide modulus μ, as well as earthquake source characteristics. An earthquake medium is modelled by plane-parallel homogeneous and isotropic layers. Express wave field in a spectral domain on a free surface from a random source of seismic waves which is inside of the I-th layer [3]:

$$U_z^{(0)} = -\widetilde{F}_2 + \frac{A}{B}\widetilde{F}_3 + \frac{M}{B}\widetilde{F}_4$$

$$U_r^{(0)} = -\widetilde{F}_1 + \frac{N}{B}\widetilde{F}_3 + \frac{A}{B}\widetilde{F}_4 \tag{3}$$

$$\overline{\widetilde{F}} = D_{i,1}^{-1} \cdot \overline{F}$$

where $\overline{F} = (F_1, F_2, F_3, F_4)^T$ - source components (steps of displacements and stresses);

$U_z^{(0)}, U_r^{(0)}$ - displacements on a free surface ;

A, B, M, N - recurrent expressions dependent on medium characteristics V_p, V_s, μ ;

$D_{i,1}$ - typical matrix measuring 4x4 of a medium over a source in the I-th layer;

$D_{i,1}^{-1}$ - inverse matrix, i.e. $D_{i,1} \cdot D_{i,1}^{-1} = E$.

D- the characteristic matrix of a n-layered medium. Thus we have obtained the solution of the direct dynamic seismic problem ,i.e. the wave field on the free surface when the medium's physical characteristics ,as well as the parameters of the source ,are known. The relationships between the components of the matrix D in Eq(3) are in a recurrent form. The author obtained them in [3-5]. For example,

$$A = d_{13}d_{31} - d_{11}d_{33} = \frac{1}{\alpha_1} S_3 P_{12}^{13} - \frac{1}{2} C_2 \cdot P\left[\overset{(n)13}{P_{13}} e^{\bar{\alpha}_1 + \bar{\beta}_1} + \overset{(n)13}{P_{24}} e^{-(\bar{\alpha}_1 + \bar{\beta}_1)} \right] -$$

$$- \frac{1}{2} S_2 \cdot \left[\overset{(n)13}{P_{14}} e^{\bar{\alpha}_1 + \bar{\beta}_1} + \overset{(n)13}{P_{23}} e^{-(\bar{\alpha}_1 + \bar{\beta}_1)} \right]; \tag{4}$$

where $\bar{\alpha}_i = kh_i(\alpha_i + \beta_i), \bar{\beta}_i = kh_i(\alpha_i - \beta_i),$

$$\alpha_i = \sqrt{1 - \frac{C^2}{V_{p_i}^2}}; \quad \beta_i = \sqrt{1 - \frac{C^2}{V_{s_i}^2}},$$

$$\overset{(n)}{P_{ek}^{mn}} = \overset{(n-1)}{P_{ek}^{12}} \overset{(n)}{a_{12}^{mn}} + \overset{(n-1)}{P_{ek}^{13}} \overset{(n)}{a_{13}^{mn}} e^{\bar{\alpha}_n + \bar{\beta}_n} + \overset{(n-1)}{P_{ek}^{14}} \overset{(n)}{a_{14}^{mn}} e^{\bar{\alpha}_n - \bar{\beta}_n} + \overset{(n-1)}{P_{ek}^{24}} \overset{(n)}{a_{13}^{mn}} e^{-(\bar{\alpha}_n + \bar{\beta}_n)} + \overset{(n-1)}{P_{ek}^{34}} \overset{(n)}{a_{34}^{mn}} + \overset{(n-1)}{P_{ek}^{23}} \overset{(n)}{a_{34}^{mn}} e^{-(\bar{\alpha}_n - \bar{\beta}_n)}$$

$$\overset{(i)}{a_{ek}^{mn}} = \overset{(i)}{a_{me}} \overset{(i)}{a_{ek}} - \overset{(i)}{a_{ek}} \overset{(i)}{a_{ek}}, \quad (i = 1, \ldots n)$$

Using the proposed the recurrent method we can determine the parameters of the source $\bar{F} = (F_1, F_2, F_3, F_4)^T$ from relationships (3) and characteristics of the medium. The author verifies this method for "the layer on the half-space", when a source of seismic waves is regarded as a horizontal circular crack and the time dependence is the same as in Brune's model, i.e.

$$F_1^{(r)}(t) = \sqrt{a^2 - r^2}(1 - e^{-\frac{t}{T}}),$$

where a is the radius of the circular crack and T is the formation time.

Using Eq(3) and Eq(4) we can find the important parameters a and T of the earthquake's focus and the characteristics of the medium V_p, V_s and μ. The results obtained were verified for the case of a two-layered model. This is shown in table 2.

Table 2. The results of the recurrent method for a two-layered model.

№	direct problem					inverse problem				
	V_p	V_s	μ	a	T	V_p	V_s	μ	a	T
	m/s	m/s	N/m²	m	c	m/s	m/c	N/m²	m	c
1	4000	2300	35 ×10⁹	10	3	4000	2300	35×10⁹	8,95	2,74
2	4500	2400	40×10⁹			4508	2415	37×10⁹		

Once the parameters of the model and the medium have been determined , the seismic moment M and the seismic wave energy E_s can be found through the known relationship.

Conclusions.

With reference to the results given in this article, the important conclusion can be drawn that the proposed method allows , on one hand , the characteristics of the model of the source to be determined mathematically accurately when the medium are known (from Eq(3)) and , on the other hand , the parameters of the models of the source and the medium to determined simultaneously. Certainly , the results obtained need to be confirmed against field data.
Additionally , the author believes that, for the modelling of a certain class of source , this approach can be introduced into applied geophysics, because modelling the sources and identifying its parameters is an important practical problem.

References

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