Radiative Friction, Critical Ruptures and Kalman-Lévy Filter

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Abstract

We summarize the progresses obtained in three research projects:

- **the regime of large velocity friction and the discovery of acoustic radiation feedback,**
- **the critical earthquake concept,**
- **the introduction of a new multiscale data-adaptative wavelet tool and the solution of the data assimilation problem (Kalman filter) in the presence of power law noise.**

Miscroscopic origin of friction: acoustic radiation

Experimental evidence of acoustic radiation feedback on sliding friction

Studies of friction are often carried out at modest relative speeds: the two moving surfaces in question typically slide past each other at 1 cm/s or much less. However, new mechanisms might appear when surfaces slide against each other at higher velocities, such as those associated with friction between tectonic plates during earthquakes. Observing the jerky "stick-slip" motion of a steel block riding on a rotating steel table, Johansen and Sornette [1] have carefully measured the friction forces for relative velocities up to 0.35 m/s, by monitoring the expansion and compression in a spring attached to the steel block. At these high velocities, we noted that the significantly increased production of sound waves (largely neglected in past analyses) dissipates a large amount of energy, stealing away some of the energy of motion required for two surfaces to slide past each other and thereby amounting to an increase in friction. This work gives direct experimental evidence of the importance of the feedback of acoustic radiation on the sliding dynamics by measuring the acoustic noise spectrum on a variety of smooth surfaces. The full noise spectrum is quantitatively explained by a simple noisy harmonic oscillator equation with a radiation damping force proportional to the derivative of the acceleration, added to a standard viscous term:

$$\ddot{x} + c_{rad} \dddot{x} + c_{vis}\dot{x} + \omega_0^2 x = \eta(t), \qquad (1)$$

where $\eta(t)$ is a "white" noise term, *i.e.*, $\langle \eta(t) \rangle = 0$ and $\langle \eta(t)\eta(t') \rangle = b\delta(t-t')$ accounting for the stochastic motion of the asperities. We have introduced two damping terms. The non-standard $c_{rad} \dddot{x}$ term is the Abraham-Lorenz expression for the first-approximation of the direct reaction force due to radiational damping. A viscous friction $c_{vis}\dot{x}$ must also be present in order to produce a friction force at constant velocities. It includes the effect of all other friction mechanisms, including a renormalisation of the Abraham-Lorenz term at the scale of individual asperities that undergo acceleration/deceleration even under constant block velocity. The comparison with experiments shows that the radiative term dominates completely above the characteristic frequency ω_0.

The observable consequence of the radiative damping term is that the amplitude spectrum exhibits an approximate $1/\omega^6$-decay for frequencies larger than the natural corner value. We expect that this finding extends to the slip-stick regime. The implication of this finding for rupture and earthquake modelling is of great potential impact. Our results suggest that the Ruina-Dieterich friction laws cannot be extended to the high velocity regime relevant for earthquakes and many cases of rupture.

This finding may illuminate the so-called "cut-off" frequency problem [2], on the source acceleration spectra of moderate to strong Californian earthquakes. The unexpected result is that the seismic spectrum falls off very fast beyond a frequency attributed to a characteristic size of cohesive fault size or to scale-length of heterogeneities of the fault plane. Our results may quantify this phenomenon by confirming the role of the radiation by accelerating asperities and specifies the quantitative shape of the spectral fall-off. However, earthquakes faults have a thick layer of fault gouge between the sliding rock blocks which might affect the magnitude of the radiation term. In order to clarify this issue, further experimental work with such an intermediate layer should be performed. Future works include the generalisation of the acoustic Abraham-Lorenz radiation damping beyond the first-order approximation and the derivation of a generalised Fluctuation-Dissipation theorem relating the noise amplitude b to the damping coefficients c_{rad} and c_{vis}.

Solid friction at high sliding velocities by an explicit 3D dynamical SPH approach

This work is motivated by the recent work of Tsutsumi and Shimamoto [3, 4], who have reached a completely novel regime, by performing friction measurements on rotation cylindrical samples at velocities up to 1.8 m/s and for slips of several tens of meters. While these results are not completely straightforward to interpret due to several experimental problems, they seem to indicate the existence of a change of regime from velocity weakening to velocity strengthening and then again to velocity weakening at the largest velocities. This last regime seems to be associated to the melting of a very thin layer.

We have developed realistic 3D numerical simulations of elastic bodies sliding on top of each other in a regime of velocities ranging from meters to tens of meters per second [5]. In this way, we probe more intimately than any exprimental setup could do the response of the bodies and the nature of the friction. We will present realistic 3D numerical simulations of elastic bodies sliding on top of each other in a regime of velocities ranging from 0.1 meters to 10 meters per second using the so-called Smoothed Particle Hydrodynamics (SPH) method. This allows us to probe intimately the response of the bodies and the nature of the friction between them. Our investigations are restricted to regimes of pressure and roughness where only elastic deformations occur between asperities at the contact surface between the slider block and the substrate. In this regime, solid friction is due to the generation of vibrational radiations which are subsequently escaping to infinity or damped out in which case energy is dissipated. We study periodic commensurate and incommensurate asperities and various types of disordered surfaces. In the elastic regime studied here, we report the evidence of a transition from zero (or non-measurable $\mu < 0.001$) friction to a finite friction as the normal pressure increases above about 10^6 Pa. For larger normal pressures (up to 10^9 Pa), we find a remarkably universal value for the friction coefficient $\mu \approx 0.06$, which is independent of the internal dissipation strength over three order of magnitudes, and independent of the detailled nature of the slider block-substrate interactions. We find that disorder may either decrease or increase μ due to the competition between two effects: disorder detunes the coherent vibrations of the asperties that occur in the periodic case, leading to weaker acoustic radiation and thus weaker damping. On the other hand, large disorder leads to stronger vibration amplitudes at local asperities and thus stronger damping. Our simulations have confirmed the existence of jumps over steps or asperities of the slider blocks occurring at the largest velocities studied (10 m/s). These jumps lead to chaotic motions similar to the bouncing-ball problem. We find a velocity strengthening with a doubling of the friction coefficient as the velocity increases from 1 m/s to 10 m/s. This reflects the increasing strength of vibrational damping.

The critical earthquake concept

Many material ruptures occur by a "one crack" mechanism and a lot of effort is being devoted to the understanding, detection and prevention of the nucleation of cracks. Exceptions to the "one crack" rupture mechanism are heterogeneous materials such as fiber composites, rocks, concrete under compression and materials and earthquakes with large distributed residual stresses. The common property shared by these systems is the existence of large inhomogeneities, that often limit the use of effective medium theories for the elastic and more generally the mechanical properties. In these systems, failure may occur as the culmination of a progressive damage involving complex interactions between multiple defects and the growing of micro-cracks. In addition, other relaxation, creep, ductile, or plastic behaviors, possibly coupled with corrosion effects, may come into play. Many important practical applications involve the coupling between mechanic and chemical effects with the competition between several characteristic time scales.

A simple realistic model of such behavior is a statistical model of earthquake foreshocks [6]. This model is based on a realistic model of dynamically evolving damage [7] which produces many growing interacting micro-cracks with an organization which is a function of the damage-stress law. Under a step-function stress loading, the total rate of damage, as measured for instance by the elastic energy released per unit time, increases on average as a power-law of the time-to-failure. In this model, rupture is a "critical point" in the statistical physics sense [8] and occurs as the culmination of the progressive nucleation, growth and fusion between microcracks, leading to a fractal network of cracks. This simple model has since then been found to describe quantitatively the experiments on the electric breakdown of insulator-conducting composites [9]. This led us to propose and test, on real engineering composite structures, the concept that failure in fiber composites is a genuine "critical" point [10]. This critical behavior may correspond to an acceleration of the rate of energy release or to a deceleration, depending on the nature and range of the stress transfer mechanism and on the loading procedure. We were thus led to propose that the power-law behavior of the time-to-failure analysis should be corrected for the presence of log-periodic modulations [10]. Log-periodicity is the hallmark of a hierarchy of discrete characteristic scales in the rupture process. Mathematically, it corresponds to adding an imaginary part to the exponent z. Intuitively, the log-periodic oscillations are oscillations that are periodic in the logarithm of the time-to-failure and thus corresponds to an accelerating frequency modulation at the critical time is approached. This acceleration of alternating ups and downs accounts for the succession of damage and quiescent phases self-organizing and culminating in the rupture.

Since the publication of Ref.[10], this method has been used extensively during our continuing collaboration with the French Aerospace company Arospatiale on pressure tanks made of kevlar-matrix and carbon-matrix composites used on the European Ariane 4 and 5 rockets. In this application, the method consists in recording acoustic emissions under constant stress rate. The acoustic emission energy as a function of stress is fitted by the above log-periodic critical theory. One of the parameters is the time of failure and the fit thus provides a "prediction" when the sample is not brought to failure in the first test [10]. The results indicate that a precision of a few percent in the determination of the stress at rupture is typically obtained using acoustic emission recorded about 20% below the stress for rupture. We now have a better understanding of the conditions, the mathematical properties and physical mechanisms at the basis of log-periodic structures [11] .

At the same time, it became tempting [12] to apply similar considerations to earthquakes. Indeed, over the years there has been a growing evidence that a significant proportion of large and great earthquakes are preceded by a period of accelerating seismic activity of moderate-sized earthquakes. These moderate earthquakes occur during the years to decades prior to the occurrence of the large or great events and over a region much larger than its rupture zone. The combined observational and simulation evidence now seems to confirm that the period of increased moment release in moderate earthquakes signals the establishment of long-wavelength correlations in the regional stress field. Large or great earthquakes appear to dissipate a sufficient proportion of the accumulated regional strain to destroy these long wavelength stress correlations. They can thus be considered as different from smaller earthquakes. According to this model, large earthquakes are not just scaled-up version of small earthquakes but play a special role as "critical points" [13, 14].

This critical-earthquake concept has been further strengthened by showing that the nature and strength of heterogeneity controls the rupture process: increasing the disorder changes rupture from first-order (abrupt) to critical (continuous with power-law properties) [15]. This was anticipated early by Mogi [16], who noticed that, for experiments on a variety of materials, the larger the disorder, the stronger and more useful are the precursors to rupture. For a long time, the Japanese research effort for earthquake prediction and risk assessment was based on this very idea. In a novel two-dimensional spring-block model of surface, inspired by the famous Burridge-Knopoff model, the stress is assumed to be released by both spring breaks and block slips [15]. This spring-block model may represent schematically for instance the experimental situation where a balloon covered with paint or dry resin is progressively inflated. An industrial application may be for instance a metallic tank with carbon or kevlar fibers impregnated in a resin matrix wrapped up around it which is slowly pressurized [10]. As a consequence, it elastically deforms, transferring tensile stress to the over-layer. Slipping (called fiber-matrix delamination) and cracking can thus occur in the over-layer. The influence of disorder has been studied: by the term "disorder", we refer to heterogeneity in material properties (elastic coefficients and rupture thresholds) as well as inhomogeneous pre-stresses. In the presence of long-range elasticity, disorder is found to be always relevant leading to a critical rupture. However, the disorder controls the width of the critical region [17] . The smaller it is, the smaller will be the critical region, which may become too small to play any role in practice. The presence of log-periodic correction to scaling in the elastic energy released has also been demonstrated numerically for the thermal fuse model [18] using a novel averaging procedure, called the "canonical ensemble averaging". Johansen and Sornette [19] have recently analyzed the acoustic emissions recorded during the pressurization of spherical tanks of kevlar or carbon fibers pre-impregnated in a resin matrix wrapped up around a thin metallic liner (steel or titanium) fabricated and instrumented by Aerospatiale-Matra Inc. These experiments were performed as part of a routine industrial procedure, which tests the quality of the tanks prior to shipment and varies in nature. It was found that the seven acoustic emission recordings of seven pressure tanks which was brought to rupture exhibit clear acceleration in agreement with a power-law "divergence" expected from the critical point theory. In addition, strong evidence of log-periodic corrections is found that quantify the intermittent succession of accelerating bursts and quiescent phases of the acoustic emissions on the approach to rupture. An improved model accounting for the crossover from the non-critical to the critical region close to the rupture point was shown to exhibit interesting predictive potential.

Ouillon and Sornette [20] have recently reported new tests of the concept that large earthquakes are "critical points" performed on rockbursts in deep South African mines. For this, the concept of an optimal time and space correlation region has been generalized and tested on the eight main shocks of a catalog provided by the South African company ISS International. In a first test, they have used the simplest signature of criticality in terms of a power-law time-to-failure formula. Notwithstanding the fact that the search for the optimal correlation size is performed with this simple power-law, they find evidence both for accelerated seismicity and for the presence of log-periodic behavior in the cumulative Benioff strain with a preferred scaling factor close to 2. They then propose a new algorithm based on a space and time smoothing procedure, which is also intended to account for the finite range and finite duration of mechanical interactions between events. This new algorithm provides a much more robust and efficient construction of the optimal correlation region, which allows them the use of the log-periodic formula directly in the search process. In this preliminary work, they have only tested the new algorithm on the largest event on the catalog. The result is of remarkable good quality with a dramatic improvement in accuracy and robustness. This confirms the potential importance of log-periodic signals. This study opens the road for an efficient implementation of a systematic testing procedure of real-time predictions.

Significant advances, both in the theoretical understanding of rupture processes in heterogeneous media and in the methodology for characterizing critical behavior have led Johansen et al. [21] to reanalyze the evidence for criticality and especially log-periodicity in the previously reported chemical anomalies that preceded the Kobe earthquake [22, 23]. The ion (Cl^-, K^+, Mg^{++}, NO_3^- and SO_4^{--}) concentrations of ground-water issued from deep wells located near the epicenter of the 1995 Kobe earthquake are taken as proxies for the cumulative damage preceding the earthquake. Using both a parametric and non-parametric analysis, the five data sets are compared extensively to synthetic time series. The null-hypothesis that the patterns documented on these times series result from noise decorating a simple power law has been rejected with a very high confidence level.

Multiscale data analysis and data assimilation in the presence of non-Gaussian noise

Data-Adaptive Wavelets and Multi-Scale Singular Spectrum Analysis

Using multi-scale ideas from wavelet analysis, we extend [24] singular-spectrum analysis (SSA) to the study of nonstationary time series, including the case where intermittency gives rise to the divergence of their variance. The wavelet transform resembles a local Fourier transform within a finite moving window whose width W, proportional to the major period of interest, is varied to explore a broad range of such periods. SSA, on the other hand, relies on the construction of the lag-correlation matrix $\mathbf{C}$ on M lagged copies of the time series over a fixed window width W to detect the regular part of the variability in that window in terms of the minimal number of oscillatory components; here $W = M\Delta t$, with Δt the time step.

The proposed multi-scale SSA is a local SSA analysis within a moving window of width $M \leq W \leq N$, where N is the length of the time series. Multi-scale SSA varies W, while keeping a fixed W/M ratio, and uses the eigenvectors of the corresponding lag-correlation matrix $\mathbf{C}^{(M)}$ as *data-adaptive* wavelets; successive eigenvectors of $\mathbf{C}^{(M)}$ correspond approximately to successive derivatives of the first mother wavelet in standard wavelet analysis. Multi-scale SSA thus solves objectively the delicate problem of optimizing the analyzing wavelet in the time-frequency domain, by a suitable localization of the signal's correlation matrix.

We present several examples of application to synthetic signals with fractal or power-law behavior which mimic selected features of certain climatic or geophysical time series. The method is applied lastly to the monthly values of the Southern Oscillation index (SOI) for 1933–1996; the SOI time series is widely believed to capture major features of the El Niño/Southern Oscillation in the Tropical Pacific. Our methodology highlights an abrupt periodicity shift in the SOI near 1960. This abrupt shift between 5 and 3 years supports the Devil's staircase scenario for the El Niño/Southern Oscillation phenomenon.

The Kalman-Levy filter: general solution of the data assimilation problem for power law noise

The Kalman filter combines forecasts and new observations to obtain an estimation which is optimal in the sense of a minimum average quadratic error. The Kalman filter has two main restrictions: (i) the dynamical system is assumed linear and (ii) forecasting errors and observational noises are taken Gaussian. In a recent work [25], we offer an important generalization to the case where errors and noises have heavy tail distributions such as power laws and Lévy laws. The main tool needed to solve this "Kalman-Lévy" filter is the "tail-covariance" matrix which generalizes the covariance matrix in the case where it is mathematically ill-defined (i.e. for power law tail exponents $\mu \leq 2$). We present the general solution and discuss its properties on pedagogical examples. The standard Kalman-Gaussian filter is recovered for the case $\mu = 2$. The optimal Kalman-Lévy filter is found to deviate substantially fro the standard Kalman-Gaussian filter as μ deviates from 2. As μ decreases, novel observations are assimilated with less and less weight as a small exponent μ implies large errors with significant probabilities. In terms of implementation, the price-to-pay associated with the presence of heavy tail noise distributions is that the standard linear formalism valid for the Gaussian case is transformed into a nonlinear matrice equation for the Kalman-Lévy filter. Direct numerical experiments in the univariate case confirms our theoretical predictions.

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