

Application of Multi-dimensional Wavelets for Visualizing Multiscale Geophysical Phenomena: Geoid Anomalies, Mixing and high Rayleigh number Convection

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Wavelets are linear mathematical transformations (e.g. Resnikoff and Wells, 1998) which can analyze both temporal signals and spatial images at different scales. The wavelet transform is sometimes called a mathematical microscope. Large wavelets give an approximate image of the signal, while smaller and smaller wavelets zoom in on small details. Until recently most of the applications of wavelets in geophysics have been focused on the use of one-dimensional wavelets to analyse time-series of the Chandler wobble or one-dimensional spatial tracks, such as topography and gravity anomalies. Recently fast multi-dimensional wavelet transforms(Bergeron et al., 1999, 2000 a and b, Yuen et al., 2000), based on second-derivatives of the Gaussian function, have been developed and they have allowed us to construct rapidly two- and three-dimensional wavelet-transforms of geophysically relevant fields, such as geoid anomalies, temperature-fields in high Rayleigh number convection , and mixing of passive heterogeneities. With wavelet transform one can view the character of a field at a particular length-scale. This continuous transform is different from orthogonal wavelet functions, which locks on to a particular hierarchy of length-scales. like a lattice. We have constructed scalograms of the two-dimensional wavelet-transformed quantities of the geoid anomalies, temperature fields in thermal convection and the scalar field depicting the heterogeneities in mixing driven by thermal convection. With these scalograms one can discern the detail changes for the different length scales ranging from long wavelength to short wavelength , ranging from about one-tenth to one-hundredth of the entire domain. We have also constructed two proxy field variables, which are associated with the maximum of the wavelet transformed quantity, E-max, and the associated local wavenumber of the wavelet, k-max. Both E-max and k-max are functions of the spatial variables and their peak functional values indicate places with sharp variations in the field values, be it the geoid, seismic velocity anomalies or the thermal fields in convection. Both mixing dynamics and thermal convection in high Rayleigh numbers display different behavior at different length-scales, as revealed by the wavelet-transformation. Long-wavelength anomalies are mixed much faster than intermediate wavelength anomalies. The thickness of the thermal boundary layer depends also on the length-scale of the wavelet interrogation.

In the geoid the ridges and subduction zones start to be discernible at around 400 km wavelength scale wavelet (see Fig. 1). Their appearance has been verified with smaller-scale wavelets down to a wavelength of around 120 km(Kido and Yuen, 2000). Many spherical harmonics are required to mimick the feature capturing capability of a single wavelet at a particular scale in the geoid problem for order of the spherical harmonic going out to 256(Kido and Yuen, 2000). Moreover, when using an adjustable wavelet with scale given by E-max of the local gravitational potential for the length-scales of 400 km, one can further detect convergent and divergent plate boundaries on the Earth's surface, which can be picked up visually with much higher resolution (less than one degree) gravity data . Other regional views show the focus of gravitational energy of short wavelength near sites of large earthquakes, such as around Turkey, Japan , Taiwan and South America (Fig. 2). The corresponding scales of the European geoid are

shown in Fig. 3. Strong focussing of geoid anomalies in the E-max quantity can be observed over Turkey. There appears to be a tantalizing phenomenological connection of these short-wavelength geoid anomalies with the recent work by Tanimoto and Okamoto (2000) on the distribution of potential energy changes induced by large earthquakes. This apparition of plate boundaries at scales of around 400 km under the wavelet microscope may indicate that there is some critical length-scale below which there is an interplay between the conversion of elastic and gravitational potential energy in the short-timescale of earthquake dynamics.

Fig. 4 shows the dynamics of mixing in Newtonian thermal convection at an effective Rayleigh number of around 10^{**6} . We see that the mixing patterns of the tracer field (Ten et al., 1997) vary greatly, depending on the length scale. The smallest scale (lower right panel) portrayed well the overall mixing in the late stage. The flows at high Rayleigh number convection , in particular at Ra close to 10^{**10} , exhibits a transition (Vincent and Yuen, 2000). In Fig. 5 we show both the E-max and k-max distributions for thermal convection with infinite Prandtl number in an axisymmetric spherical shell. More than two million finite-difference grid points have been employed in this calculation of a basally-heated configuration . The E-max and k-max distributions yield complementary information. The focussing of the small upwellings is brought out clearer with E-max, while k-max distribution shows the local dynamics much better. Finally Fig. 6 compares the original temperature field (upper-left panel) with the temperature field at various scales, varying from small scale(upper-right panel) to the extremely small (lower-right panel). At the extremely scales we must resort to zoom-in techniques for resolving the local features. This aspect stresses the need of high-resolution display device in visualizing the intricate dynamics of large-scale numerical simulations.

In sum, we have demonstrated here the possibilities of using multidimensional wavelets in looking at the multiscale features of several interesting geophysical problems. We believe that these same techniques can be brought to bear on analyzing the dynamics in earthquake physics, such as the time-dependent strain field from INSAR images , the time-dependent gravity field from the coming GRACE mission and high-resolution elastodynamic rupture calculations with realistic constitutive relationships.

References

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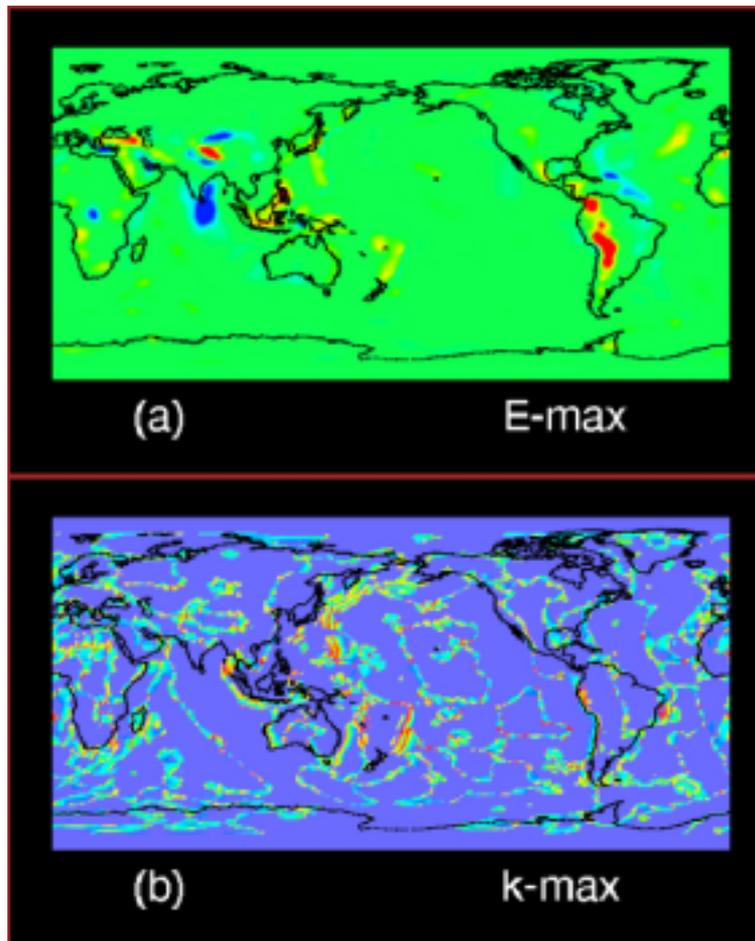


Figure 1.

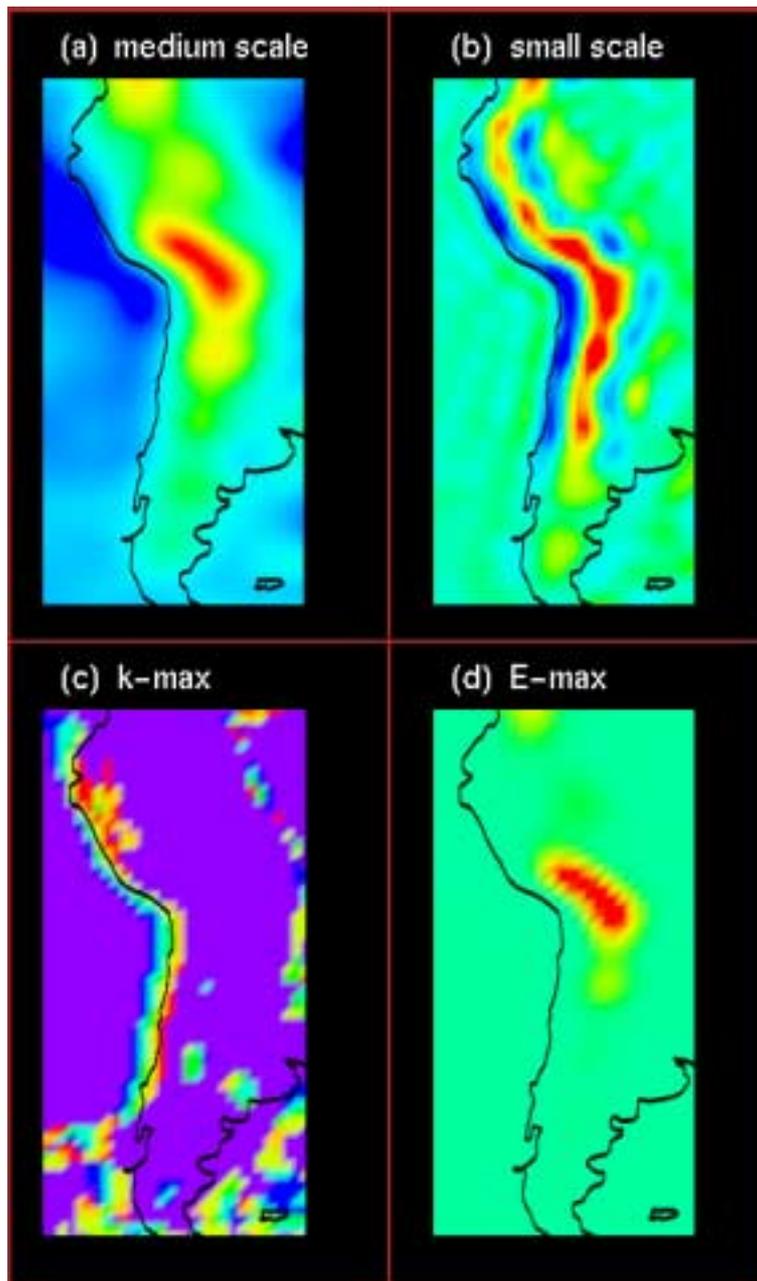


Figure 2.

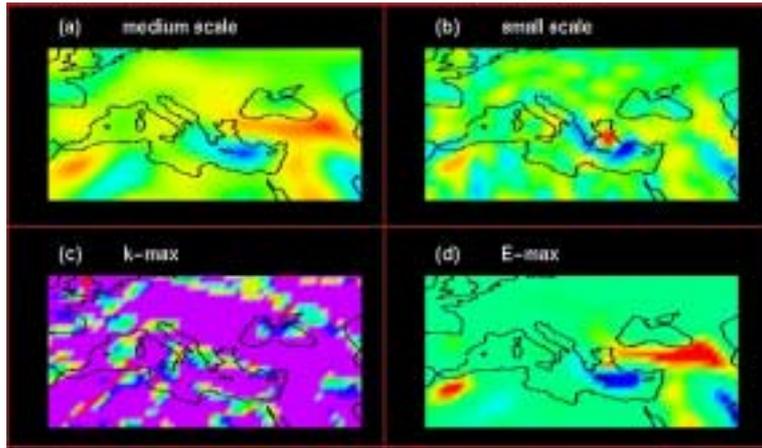


Figure 3.

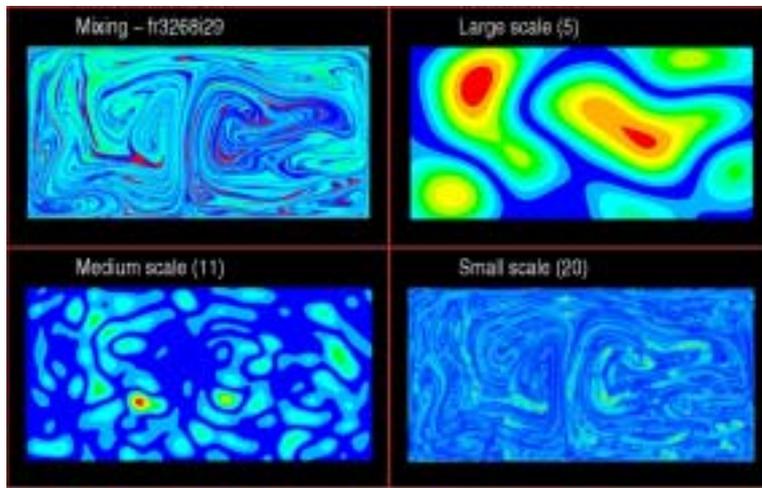


Figure 4.

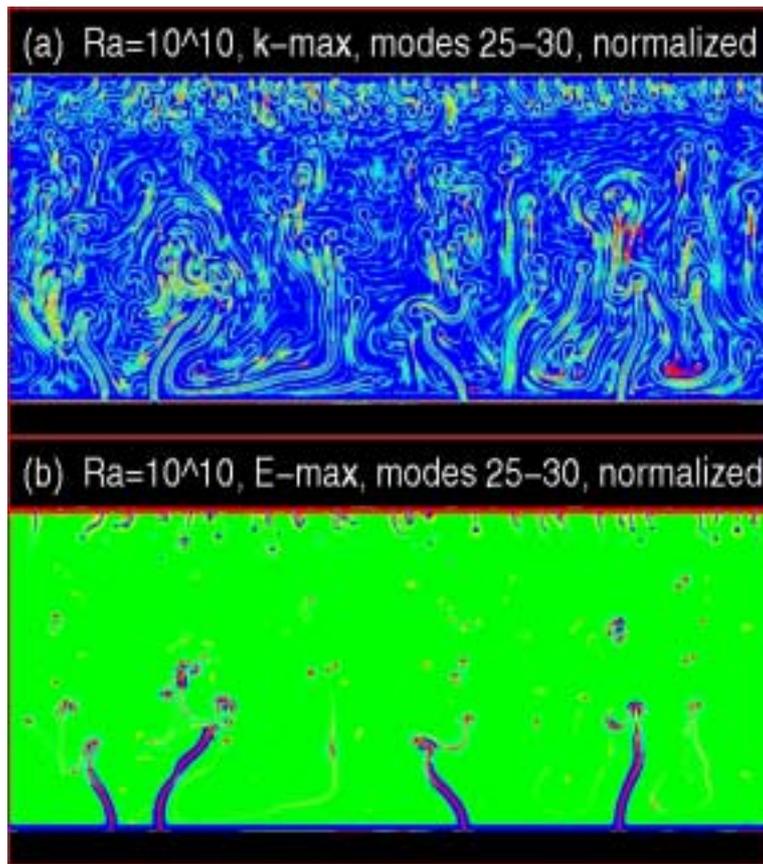


Figure 5.

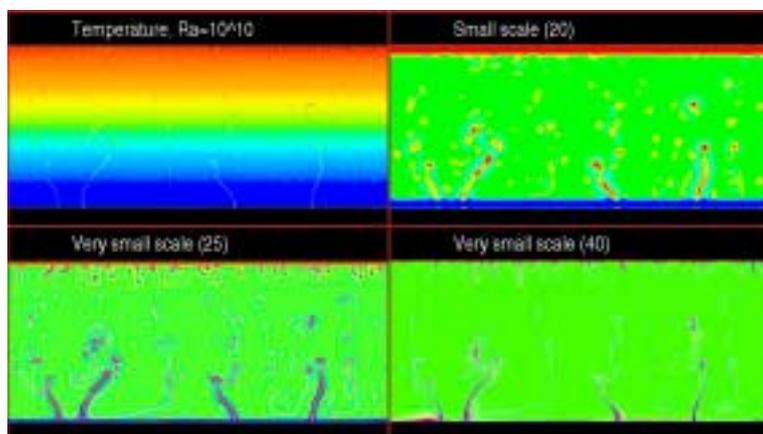


Figure 6.